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Approximate solution of non-linear dynamic energy model for multiple effect evaporator using fourier series and metaheuristics

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Abstract−This article presents the approximate solution of non-linear dynamic energy model of multiple effect evaporator (MEE) using Fourier series and metaheuristics. The dynamic model of MEE involves first-order simultaneous ordinary differential equations (SODEs). Prior to solving the dynamic model, the non-linear steady-state model is solved to obtain the optimum steady-state process parameters. These process parameters serve as the initial conditions (constraints) for the SODEs. The SODEs are exemplified as an optimization problem by the weighted residual function to produce their approximate solutions. The optimization task is to find the best estimates of unknown coefficients in the Fourier series expansion using two preeminent metaheuristic approaches: Particle swarm optimization and harmony search. Besides, the influence of the number of approximation terms in Fourier series expansion on the accuracy of the approximate solutions has been investigated. The solution of the dynamic model assists in the investigation of open-loop dynamics of the MEE. Moreover, the acquired results may assist in designing suitable controllers to ensure energy-efficient performance of MEE and to monitor the product quality. The optimization results reveal that both the metaheuristic approaches offer minimum violation of the constraints and, therefore, validate their efficiency in solving such complex non-linear energy models.

Keywords: Multiple Effect Evaporator, Non-linear Steady-state Model, Dynamic Model, Metaheuristics, Particle Swarm Optimization, Harmony Search

INTRODUCTION

Multiple effect evaporators (MEEs) are employed in the evaporative process at paper mills with the purpose of enhancing the concentration of weak black liquor (BL) for its utilization as a biofuel [1]. The assurance of optimum energy efficiency and an effective control of MEE necessitate an inclusive knowledge of system behavior in the dynamic state [2]. In addition, the system behavior in the steady-state is substantially directed towards the investigation of system dynamics for the design of efficient controller based on the system transience. Thus, meticulous information of the system behavior puts emphasis on an in-depth knowledge and understanding of the steady-state and dynamic model solutions of MEE.

The zone of mathematical modeling and simulation of MEEs has received much attention over the past few decades [3-6]. In quest of various steady-state process parameters for energy-efficient performance of MEE, various approaches have been employed to solve the steady-state mathematical model of MEE encompassing simultaneous non-linear algebraic equations. These include classical numerical techniques based on iterative methods [7,8], a dynamic programming approach (Interior-Point Methodology, I-PM) [9] and soft computing approaches [10,11]. On account of the erroneousness and ineffectiveness of the prevailing numerical techniques in quest of the exact and analytical solutions of a system of non-linear algebraic equations, research has tended to focus on the robustness and efficiency of various metaheuristic techniques for evaluating the near-optimal solutions. Various soft computing techniques have been employed to solve the non-linear steady-state energy model of MEE [12,13]. Moreover, various metaheuristic approaches such as particle swarm optimization (PSO) [14,15] and harmony search (HS) [16-19], have been utilized to solve simultaneous non-linear algebraic equations. However, their performance has been tested merely on a few benchmark functions.

Now, as far as the dynamic model of MEE is concerned, most of the previous works are limited to the evaluation of system transfer functions using the linearization of the dynamic model and have failed to address the issue of obtaining approximate solutions [2, 20,21]. Thus, a derelict zone in the field of dynamic model simulation of MEE is the accomplishment of the model solutions, which has not yet been established in the existing literature. Nevertheless, the dynamic model of MEE, typically involving a system of ordinary differential equations, can be solved by efficacious utilization

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of various numerical approximation techniques. However, these numerical techniques suffer from a number of pitfalls. As a result of the operational constraints and restrictions in each of these numerical techniques, strict confinements on their functioning domains exist. Moreover, the time-consuming and labor-intensive computations engulfed by these classical numerical techniques have been clearly recognized as their major shortcomings. Additionally, these techniques are frequently infeasible, impractical [22,23] and inexact. In the context of assessing the near-optimal solutions to real-valued complex numerical problems along with realistic computational time, the competencies of various metaheuristic approaches have been demonstrated, wherein the exact and analytical problem-solving approaches are incapable of producing better results [24-27]. In recent years, there has been growing interest in the utilization of Fourier series expansion (as the approximate base function) along with metaheuristic algorithms towards the estimation of approximate solutions of ODEs [22,28]. This approximation method offers an acceptable solution for any complex type of ODEs for higher orders in their implicit forms. It has been widely used due to its simple implementation and straightforward approach to solve different types of linear and non-linear ODEs. Some of the metaheuristic techniques, for instance, particle swarm optimization (PSO) [22,23,29], cuckoo search algorithm [22], water cycle algorithm [22], genetic algorithms (GAs) [30,31], genetic programming [32], and others [33,34] have corroborated their proficience and competence to solve ODEs as well as optimizing the Fourier series coefficients, thereby yielding their approximate solutions. However, the solution of ODEs involved in MEE using Fourier series along with metaheuristic techniques still needs to be explored and proves to be a vibrant research area seeking further investigation.

As mentioned earlier, higher order simultaneous non-linear ODEs can be solved effectively by Fourier series in comparison to classical approaches. However, the effectiveness of the Fourier series lies behind the optimal values of Fourier coefficients. Therefore, metaheuristic techniques are used to estimate the Fourier coefficients in order to reach the optimal solution of dynamic model of MEE. In the present analysis, a well-known MEE system, heptad's effect evaporator (HEE) working in the backward feed flow configuration (BFFC), set up in a North Indian Pulp and Paper Mill (at Saharanpur, Uttar Pradesh, India) has been considered. The non-linear steady-state model of MEE house embraces a set of fourteen equations: simultaneous non-linear algebraic equations (SNLAEs) with fourteen unknown process variables (6-vapor temperatures (T_i, i=2-7), 7-liquor flow rates (L_i, i=1-7) and overall steam consumption (SC) of the MEE house (V_1)). The solutions of these SNLAEs are the steady-state process parameters which are acquired by transforming the steady-state model into optimization model. The formulated optimization model is then optimized by aforesaid metaheuristic approaches (PSO and HS) in the pursuit of the optimum steady-state process parameters. Further, steam economy (SE), an energy-efficiency (performance) parameter of MEE, is computed using the optimal estimates of unknown process parameters. Subsequently, the dynamic model entailing fourteen ODEs, *i.e.*, simultaneous ordinary differential equations (SODEs), is solved using Fourier series expansion. For solving the dynamic model of MEE house, the SODEs of the dynamic model are represented as an optimization problem by making use of the elementary theories of Fourier series expansion and aforementioned metaheuristic approaches (PSO and HS). The initial conditions of these SODEs are the steady-state process parameters (acquired by solution of non-linear steady-state energy model) of the MEE house. The weighted residual function has been chosen as the performance index to be minimized by utilizing the optimal estimates of Fourier series coefficients. Here, the unknown coefficients of Fourier series are treated as the design variables on the lookout for approximate solutions of SODEs. Consequently, variations in the concentration of black liquor along with the temperature of vapor generated at each effect of MEE with respect to time are obtained as the solutions of the dynamic model of MEE. Moreover, the effect of variation in the number of approximation terms of Fourier series expansion in terms of accuracy has been scrutinized. The application of the above-mentioned metaheuristic techniques demonstrates their efficiency and robustness as a first attempt aimed towards solving such a complex SNLAEs and SODEs.

Indeed, the novelty of the present work lies in solving the non-linear dynamic energy model of MEE using nature-inspired algorithms and Fourier series expansion. Conclusively, the following are the key contributions of the present investigation:

1. To search the optimal solution of non-linear steady energy model of MEE by metaheuristic approaches to ensure its energy-efficient performance.
2. To compare the energy performance parameters (SE and SC) obtained by PSO and HS with real-time plant estimates and previous works.
3. To solve the dynamic model of MEE through Fourier series and optimize the Fourier coefficients using metaheuristic approaches (PSO and HS) for studying the system dynamics.
4. To investigate the effect of variation in number of approximation terms in Fourier series on the accuracy of the model solution.

The present investigation makes use of MATLAB (R2016a) programming environment for carrying out the simulation tasks on an AMD A8-7410 APU with AMD Radeon R5 Graphics (2.20 GHz and 4.00GB RAM) on a 64-bit OS, x64-based processor. Throughout this paper, we use the terms 'HEE' and 'MEE' interchangeably, in accordance with the conventional practice and usage.

The organization of the rest of this article is in the following fashion. Section 2 provides details about the development of non-linear steady-state and dynamic mathematical models of HEE System for BFFC. Section 3 provides a brief discussion of the optimization strategies utilized in this work for solving the steady-state and dynamic models. The aforementioned metaheuristic approaches have been briefly explained in this section. Section 4 describes the methodology for solution of these mathematical models by formulation of the objective function. Section 5 and 6 present the results and discussion. Our concluding remarks are presented in Section 7.

DEVELOPMENT OF ENERGY MODELS OF MEE: A CASE STUDY OF PULP AND PAPER MILL

1. Process Description

Earlier literature reports that the BFFC of MEE is more appro-

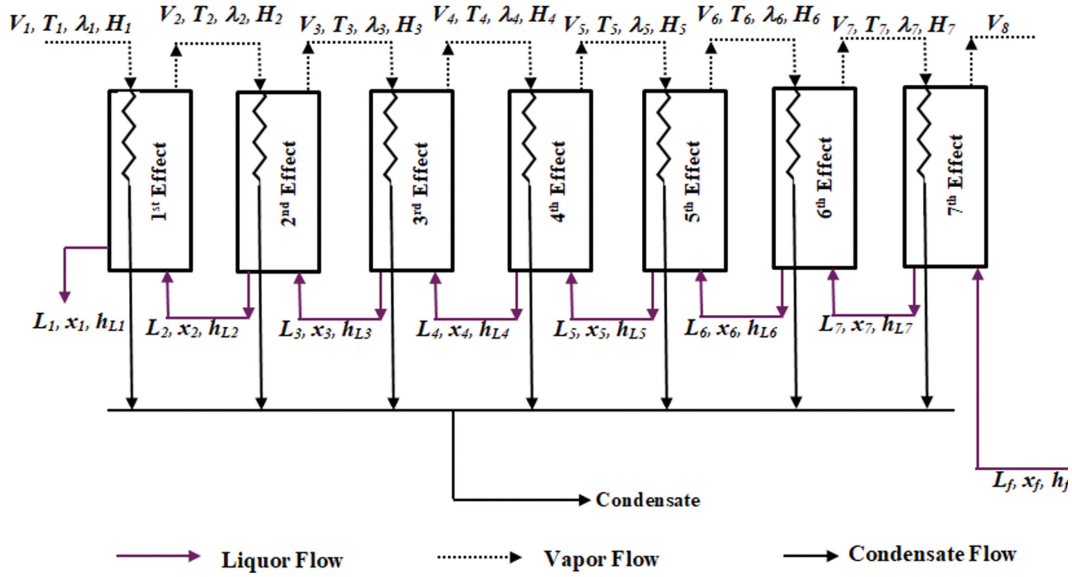

Fig. 1. Layout of a heptads' effect evaporator in BFFC.

priate relative to the forward-feed and mixed-feed configurations on account of its optimal energy performance parameters (steam economy and steam consumption) [2,9]. Within this framework, the present analysis examines the heptads' effect falling film evaporator operating in the BFFC to inspect the steady-state and dynamic response of the MEE through mathematical modeling. A typical sketch for HEE based falling film evaporator functioning in the BFFC is illustrated in Fig. 1.

Fig. 1 indicates the relation between various effects wherein the effects are numbered from left to right as 1 to 7 in a successive manner. Here, V_i and T_i represent the flow rate and temperature of vapors entering the overhead of the i^{th} effect, respectively. x_i (i=1-7) represent the respective concentrations of liquor acquired from the i^{th} effect. As depicted in Fig. 1, the BFFC of the HEE involves the flow of weak BL (with flow rate L_f, concentration x_f and temperature T_f) to the last (7^{th}) effect and the input steam (with flow rate V_1) to the 1^{st} effect of the MEE system. The flow of weak BL from the 7^{th} to the 1^{st} effect is in a sequential manner and countercurrent to the input steam flow. Thus, the concentrated BL is acquired from the 1^{st} effect. This arrangement consumes the vapor acquired at each effect as a source of heat for the successive effects. The vapor generated in 1^{st} effect (V_2) serves as a heat source for the 2^{nd} effect, and the vapor generated in the 2^{nd} effect (V_3) is fed as a source of heat to the 3^{rd} effect, and so on, as evident from Fig. 1.

A thorough understanding of the system dynamics is indispensable for the design of an accurate and appropriate control algorithm for effective set-point tracking and disturbance rejection. To study the system dynamics, it is mandatory to ascertain steady-state parameters for solving the dynamic model of the system. Henceforth, in advance of analyzing the system transience, steady-state analysis has been performed, details of which are given in the next sub-section.

2. Model Development in Steady State

With the intention of developing the steady-state mathematical model for HEE, the basic mass, component, energy and heat transfer balance equations are applied to each effect of MEE. For N number of effects of an HEE system, 2N non-linear equations are obtained which are highly governed by the unknown process parameters: the amount of fresh steam supplied V_1, the flow rate of BL (L_i, i=1-7) and temperatures of generated vapor at various effects (T_i, i=2-7). Consequently, the considered HEE system yields fourteen non-linear equations with equal number of unknown operating variables to be estimated.

Few assumptions are made under ideal conditions for the development of SNLAEs for model formulation of MEE. These assumptions are made to avoid the real-time complexities and for lumping the process parameters and are enumerated below:

- Negligible loss of heat to the surroundings.
- Negligible deviation in composition and boiling point elevation in each effect.
- Negligible foaming of black liquor.
- Negligible fouling and scaling effects.
- Thermal equilibrium between black liquor and vapor produced at each effect.

Besides the abovementioned assumptions, the simulation of MEE also entails some useful thermo-physical correlations including latent heat of vaporization (λ), enthalpy of vapor produced (H) and enthalpy of BL (h_L) at each effect as reported in [9]. The non-linear dependency of these parameters on vapor temperature and concentration of BL results in a non-linear mathematical model. These correlations (Eq. (1)-(3)) are exploited afterwards en route for thorough modeling of HEE.

$$\lambda_i = -0.003857T_i^2 - 2.069T_i + 2497 \quad (1)$$

$$H_i = -0.0002045T_i^2 + 1.677T_i + 2507 \quad (2)$$

$$h_{L_i} = 4.187(1 - 0.54x_i)T_{L_i} \quad (3)$$

Previous literature reports the development of non-linear steady-state model for MEE [9] which is discussed here comprehensively. For energy optimization of MEE, the SNLAEs for the developed

Table 1. Operating process parameters of heptads' effect based MEE

Operating process parameter (s)	Value (s)
Total number of effect (s)	7
Temperature of weak BL to be fed (°C)	65
Concentration of weak BL to be fed, x_f	0.118
Feed flow rate of weak BL to be fed, L_f (kg/s)	15.611
Vapor temperature generated at 7th (last) effect (°C)	52
Effective heat transfer area: A_1 and A_2, A_3–A_6, and A_7 (m^2)	540, 660 and 690
Enthalpy of weak BL to be fed, h_f (kJ/kg)	254.81
Mass hold up for each effect, M_i (kg)	0.833

Table 2. Steady-state values of Heat transfer coefficients and liquor concentrations of HEE

Parameter	Effect 1	Effect 2	Effect 3	Effect 4	Effect 5	Effect 6	Effect 7
U_i	0.165	0.296	0.985	1.08	1.676	1.792	2.369
x_i	0.5286	0.4321	0.3285	0.2465	0.1954	0.1625	0.1394

non-linear model have been exploited and simulated in the present study. The preliminary model formulation begins with the establishment of the energy balance equations for each MEE effect, bearing in mind the energy inflow into and outflow from the steam chest of each effect of the MEE. Establishing the energy balance equation for the first effect of the MEE:

Energy due to the liquor entering from 2nd effect with sensible heat + Energy due to input steam entering the vapor chest of 1st effect = Energy owing to the vapor leaving the 1st effect + Energy owing to the liquor leaving the 1st effect

which yields:

$$V_1\lambda_1+L_2h_2=L_1h_1+V_2H_2 \tag{4}$$

The total material balance around the 1st effect yields the flow rate of vapor produced in 1st effect as $V_2=L_2-L_1$. Thus, Eq. (4) can be rewritten as:

$$V_1\lambda_1+L_2h_2=L_1h_1+(L_2-L_1)H_2 \tag{5}$$

The latent heat supplied by fresh input steam and the overall amount of heat transferred to the 1st effect are equated to yield Eq. (6),

$$U_1A_1(T_1-T_2)=V_1\lambda_1 \tag{6}$$

By the same token, the flow rate of vapor produced at i^{th} effect (*i.e.*, V_i for i=2-6) can be computed by the total material balance around the i^{th} effect which gives: $V_{i+1}=L_{i+1}-L_i$. Accordingly, for effects i=2-6, the balance equations can be generalized by Eqs. (7)-(8) as:

$$(L_i-L_{i-1})\lambda_i+L_{i+1}h_{i+1}-L_ih_i-(L_{i+1}-L_i)H_{i+1}=0 \tag{7}$$

$$U_iA_i(T_i-T_{i+1})-(L_i-L_{i-1})\lambda_i=0 \tag{8}$$

In the same way, for the 7th effect, the balance equations can be written as given by Eqs. (9)-(10):

$$(L_7-L_6)\lambda_7+L_fh_f-L_7h_7-(L_f-L_7)H_8=0 \tag{9}$$

$$U_7A_7(T_7-T_8)-(L_7-L_6)\lambda_7=0 \tag{10}$$

Finally, fourteen non-linear algebraic equations (SNLAEs) are acquired, resulting in the steady-state model formulation as demonstrated by Eqs. (5)-(10). In the present study, the steady-state analysis has been carried out by the operational data presented in Table 1 [2,9] which is picked up from the aforementioned paper mill. Moreover, the steady-state values of U_i and x_i for i=1-7 as acquired from real-time plant estimates are also utilized for simulation and are presented in Table 2. The steady-state values of U_i and x_i are utilized in solving the steady-state model (since the SNLAEs include the terms U_i and x_i). Further, the steady-state values of liquor concentrations at each effect serve as the initial conditions for x_i in the SODEs of the dynamic model of MEE (discussed later in Section 4).

3. Model Development in Dynamic State

This section deals with the development of the dynamic model for HEE system based on the research [2,20,21]. The dynamic model of HEE entails a set of fourteen differential equations of the first-order obtained under similar aforementioned conditions utilized for steady-state model formulation.

Mass balance

$$\text{At effects 1-6: } \frac{dM_i}{dt}=L_{i+1}-L_i-V_{i+1}, \text{ for i =1 to 6} \tag{11}$$

$$\text{At effects 7: } \frac{dM_7}{dt}=L_f-L_7-V_8 \tag{12}$$

Total mass component balance

$$\text{At effect 1: } \frac{d(M_1x_1)}{dt}=L_2x_2-L_1x_1 \tag{13}$$

$$\text{which on simplification gives: } \frac{dx_1}{dt}=\frac{L_2(x_2-x_1)+x_1V_1}{M_1} \tag{14}$$

$$\text{similarly, for effects 2-6: } \frac{dx_i}{dt}=\frac{L_{i+1}(x_{i+1}-x_i)+x_iV_i}{M_i}, \text{ for i =2 to 6} \tag{15}$$

$$\text{At the last effect: } \frac{dx_7}{dt} = \frac{L_f(x_f - x_7) + x_7 V_7}{M_7} \quad (16)$$

Total energy balance

$$\text{At effect 1: } \frac{d[M_1 h(T_1, x_1)]}{dt} = L_2 h(T_2, x_2) + V_1 \lambda(T_s) - L_1 h(T_1, x_1) - V_2 H(T_1) \quad (17)$$

$$\text{on solving Eq. (13), we obtain: } \frac{d[h(T_1, x_1)]}{dt} = \frac{L_2[h(T_2, x_2) - h(T_1, x_1)] + V_1 \lambda(T_2) - V_2[H(T_1) - h(T_1, x_1)]}{M_1} \quad (18)$$

$$\text{Similarly, for effects 2-6: } \frac{d[h(T_i, x_i)]}{dt} = \frac{L_{i+1}[h(T_{i+1}, x_{i+1}) - h(T_i, x_i)] + V_i \lambda(T_{i-1}) - V_{i+1}[H(T_i) - h(T_i, x_i)]}{M_i}, \quad (19)$$

for i=2 to 6

$$\text{At effect 7: } \frac{d[h(T_7, x_7)]}{dt} = \frac{L_f[h(T_f, x_f) - h(T_7, x_7)] + V_7 \lambda(T_6) - V_8[H(T_7) - h(T_7, x_7)]}{M_7} \quad (20)$$

Using the thermo-physical correlations presented in Eq. (1)-(3), the above equations can be re-written as:

$$\text{For effect 1: } \frac{dT_1}{dt} = \frac{[\{L_2(4.187 - 2.26098x_2)(T_2 - T_1)\} + V_1 \lambda(T_2) + V_2(4.187T_1 - H(T_1))]}{M_1(4.187 - 2.26098x_1)} \quad (21)$$

$$\text{Similarly, for effects 2-6: } \frac{dT_i}{dt} = \frac{[\{L_{i+1}(4.187 - 2.26098x_{i+1})(T_{i+1} - T_i)\} + V_i \lambda(T_{i-1}) + V_{i+1}(4.187T_i - H(T_i))]}{M_i(4.187 - 2.26098x_i)} \quad (22)$$

$$\text{At effect 7: } \frac{dT_7}{dt} = \frac{[\{L_f(4.187 - 2.26098x_f)(T_f - T_7)\} + V_7 \lambda(T_6) + V_8(4.187T_7 - H(T_7))]}{M_7(4.187 - 2.26098x_7)} \quad (23)$$

Thus, the dynamic model of HEE involves the above simultaneous ordinary differential equations (SODEs) of the first-order with concentrations (x_i, i=1-7) and vapor temperatures (T_i, i=1-7) as dependent variables, 'time' being the independent variable.

METAHEURISTIC APPROACHES FOR OPTIMIZATION

For carrying out the optimization tasks, the present analysis makes use of two metaheuristic approaches: PSO and HS. This section offers a narrow sketch and short discussion of these approaches. However, more valuable insights in context of these approaches are accessible in the extensive available literature.

1. Particle Swarm Optimization (PSO)

Particle swarm optimization (PSO) is a stochastic metaheuristic optimization technique inspired by nature. PSO carries out the inspection and survey of the search space by a tuning-based approach in order to tune the trajectories of the individual particles [35]. The estimation of the overall best solution in the midst of all the prevailing best solutions is the central goal of PSO algorithm for solving any optimization problem. This is accomplished by performing the optimization task for a predefined number of iterations or by observing the improvement in the value of the objec-

Algorithm 1. Particle Swarm Optimization (PSO)

begin
Define the Objective function J(z)
Set the parameters of PSO algorithm
- Population size (Q=100)
- Inertial weight w (w_{max}=0.9, w_{min}=0.4)
- Acceleration factors [personal and global] (c_1, c_2: Default set to 2)
- Maximum Iterations (maxIter=1000)

Initialize locations z_q and velocity v_q of Q particles.
while (stopping criterion)
Evaluate the fitness of particles $F_q^k = J(z_q^k)$; ∀ q and find the index of the best particle (b)
Choose P $best_q^k = z_q^k$; ∀ q and $Gbest_q^k = z_b^k$
$w = w_{max} - \frac{k(w_{max} - w_{min})}{maxIter}$
Update velocity and position of particles using
$v_{q,j}^{k+1} = (w \times v_{q,j}^k) + (c_1 \times r_1 \times (Pbest_{q,j}^k - z_{q,j}^k)) + (c_2 \times r_2 \times (Gbest_{q,j}^k - z_{q,j}^k))$
$z_{q,j}^{k+1} = v_{q,j}^k + z_{q,j}^k$
Estimate the fitness $F_q^{k+1} = J(z_q^{k+1})$; ∀ q and locate the index of the best particle
Update Pbest and Gbest if there is an increase in the fitness
end while
return the best solution (Gbest)
end

Note: r_1 and r_2 denote random values lying between 0 and 1

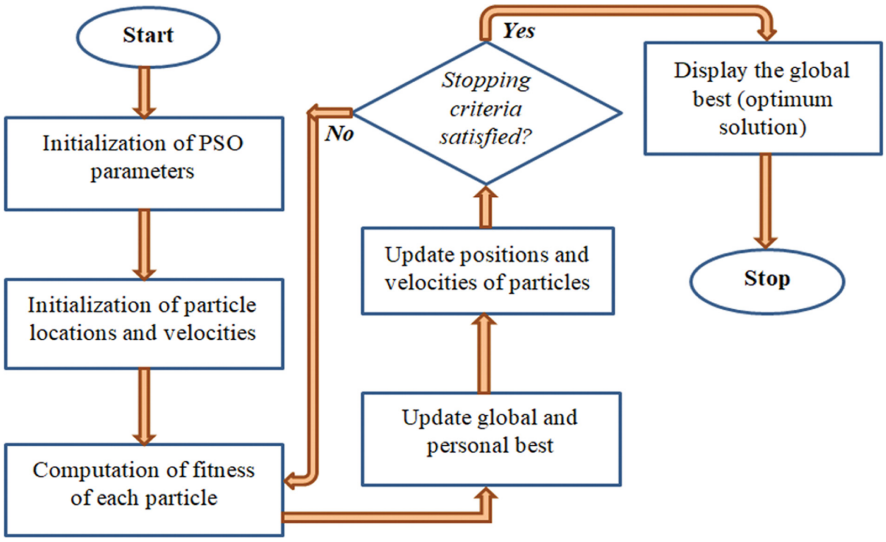

Fig. 2. Outline of PSO Algorithm for function optimization.

tive function [14,27]. Here, Algorithm 1 presents the generalized framework of PSO algorithm. Additionally, an outline of the PSO algorithm (as depicted in Fig. 2) has also been presented for better understanding. A comprehensive discussion of PSO algorithm is beyond the scope of this article. However, a profound elucidation in this context can be retrieved from a massive amount of research articles.

2. Harmony Search (HS) Algorithm

Harmony search (HS) algorithm is a stochastic population-based metaheuristic approach for solving a variety of optimization problems. HS algorithm imitates the process of music improvisation [36]. In general, in quest of the impeccable state of a splendid harmony and for aesthetic refinement, the musician searches the best harmony accompanied by harmony refinement process. In a simi-

Algorithm 2. Harmony Search (HS) Algorithm

begin
Define the Objective function
Set the parameters of HS algorithm
- D (number of decision/design variables=14 for steady-state model solution)
- HMS (Harmony Memory Size, *i.e.*, number of solution vectors in harmony memory=30)
- HMCR (Harmony Memory Consideration Rate=0.9)
- PAR (Pitch Adjustment Rate=0.1)
- b* (Distance Bandwidth)
- Max_Imp (Maximum number of Improvisations, i.e., total number of function evaluations=1000)

Create Harmony Memory (HM) by producing random harmony vectors
while (k<Max_Imp)
  while (d≤D)
    if (rand*<HMCR),
      Choose a random value from HM for the variable d
      if (rand*<PAR),
        Slightly modify the pitch within the distance bandwidth
      end if
    else
      Randomly choose a value (any pitch within bounds)
    end if
  end while
  Take the new solution (harmony) if it is better
end while
return the current best solution (best harmony)
end

*b and rand denote random values lying between 0 and 1

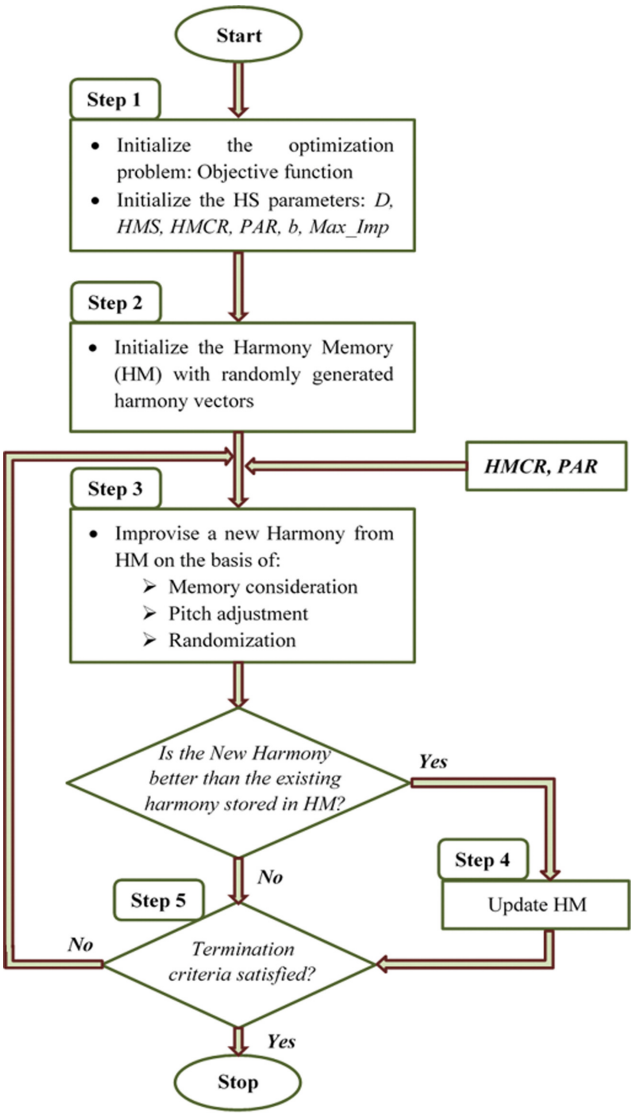

Fig. 3. Sketch of HS Algorithm for function optimization.

lar manner, by utilizing the random initial guesses or past values of the design variables, the optimum solution of any optimization model is discovered by HS algorithm [17,18]. With the intention of eluding the needless increase in the length of this article, thorough explanation of HS algorithm has not been provided here. However, a brief overview of HS algorithm has been elucidated in this work as given in Algorithm 2. Furthermore, a rough idea of HS algorithm has been illustrated in Fig. 3.

MODEL SOLUTION

This section explains the methodology for the solution of the complex non-linear steady-state model and dynamic model of the HEE system under consideration.

1. Solution of Steady State Model: Energy Optimization of HEE

With an eye to optimizing the energy efficiency of the HEE system utilized in the Kraft Recovery process in paper mills, first, the objective function **J(z)** is formulated. This objective function is chosen with the aim of estimating the optimum values of the decision variables **z** *i.e.*, V_1, L_i (i=1-7) and T_i (i=2-7). As a result, the process parameters and overall performance of the MEE system are optimized.

1-1. Formulation of Objective Function: Transforming SNLAEs to Optimization Problem

To formulate the objective function **J(z)**, the present investigation transforms the SNLAEs developed for the HEE system into an optimization problem. The merit function **J(z)** is taken as the summation of the square of non-linear equations and is given by Eq. (24).

$$J(z)=\sum_{e=1}^{14} f_e^2(z) \tag{24}$$

Here, $f_e(\mathbf{z})$ (e=1-14) represent the non-linear algebraic equations developed for each effect of the MEE system by equating the energy balance equations to zero. Therefore, **J(z)** is the anticipated objective function to be optimized with the purpose of searching the optimum values of the unknown process parameters, **z**.

For the BFFC of HEE put forward in this work, the conditions for bounds of vapor temperature at each effect, T_i (i=2-7), are to be defined. As presented in Table 1, the produced vapor from the last effect is held at a constant temperature of 52 °C (T_7) and also the live steam has been supplied at a fixed temperature of 147 °C (T_s). The temperature of vapor generated at first effect (T_1) has been set to 147 °C. For maximum heat transfer through MEE effect, the temperature of the earlier effect should be higher than later effect. For that reason, to realistically match up with the steady drop in temperature T_i from the first effect to the last effect, the feasible constraints are given by Eq. (25). For the steam consumption (SC) *i.e.*, the flow rate of fresh steam supply to the MEE house, a feasible bound for V_1 (in kg/s) has been chosen in the range of [0:3]

$$T_i > T_{i+1} \text{ (i=1-6) and } T_7 > 52 \ (T_8) \tag{25}$$

To achieve maximum energy efficiency of MEE, each effect of the MEE may possibly operate at optimal values of input steam flow rate (V_1), liquor flow rate (L_i) and vapor temperature (T_i). The objective function **J(z)** is optimized by PSO and HS in the hunt for these optimum parameters, *i.e.*, the decision variables **z**. For this, the appropriate realistic bounds taken from real-time operational data from the aforesaid paper mill and from earlier reported literature work [2,9] are utilized. A detailed description of these bounds (boundary conditions) on generated vapor temperatures (T_i) and liquor flow rates (L_i), respectively, for the HEE in BFFC is given in Table 3.

Table 3. Lower and Upper bounds of temperature of vapor generated and exit liquor flow rate for i^{th} effect of HEE

Vapor temperatures	T_i (°C) ∈ [100 : 110; 70 : 85; 66 : 74; 60 : 70; 55 : 65; 52 : 63], (i=2-7)
Liquor flow rates	L_i (kg/s) ∈ [2 : 5; 3.5 : 6; 4.5 : 7; 6.5 : 9; 9 : 11; 10.5 : 13; 13 : 15], (i=1-7)

1-2. Computation of Steady-state Process Parameters

Using the MATLAB programming environment, the formulated optimization problem **J(z)** is optimized by means of metaheuristic approaches. To end with, the optimum performance parameter, SE, is computed by exploiting the optimum values of the decision variables. SE is a performance parameter which governs the energy-efficiency of the MEE. Clearly, the steady-state model solution yields optimal estimates of unknown process parameters. These parameters include liquor flow rates, vapor temperatures and the overall steam consumption (SC). SC (in kg/s) is the amount of fresh steam input to the MEE (denoted by V_1, as shown in Fig. 1). For the HEE based MEE system, SE refers to the aggregate amount of vapor produced from all the seven effects for a given amount of steam input (SC). It is mathematically given by Eq. (26). It can be observed that SE is a unitless quantity and an inverse relationship exists between SE and SC. Clearly, the optimum SE is evaluated by utilizing the optimum estimates of L_1 and V_1.

$$SE = \frac{\sum_{i=2}^{8} V_i}{V_1} = \frac{L_f - L_1}{V_1} \tag{26}$$

The present analysis investigates the performance of PSO and HS by solving the formulated optimization problem **J(z)** for the MEE system by means of 20 independent runs. This yields the optimal values of the process variables, $\mathbf{T}_i$, $\mathbf{L}_i$ and $\mathbf{V}_1$, in the feasible operating range and bounds. After that, these optimum process parameters are exploited for the computation of SE. Fig. 4 illustrates a self-explanatory summary of the solution procedure for the estimation of optimum steady-state process parameters of MEE.

2. Solution of Dynamic Model

The dynamic model of MEE embraces a set of fourteen ordinary differential equations, SODEs; and recently, the concept of Fourier series expansion has been used as a base approximate function for finding the approximate solution of ODEs [23]. With the reference of the novel work carried out in [23], a remarkable work has been demonstrated in order to solve 10-ODE problems, including mechanical vibration problems and further, tested on more ODE problems using weighted residual method and hence, improve the approach efficiency [37]. In the present work, the superiority of using new weight function (unit weight) is shown and two metaheuristic algorithms (the PSO and HS) are employed for optimization phase. The solution approach for solving ODEs is essentially the same as that utilized in [22,23] and is explained in this section.

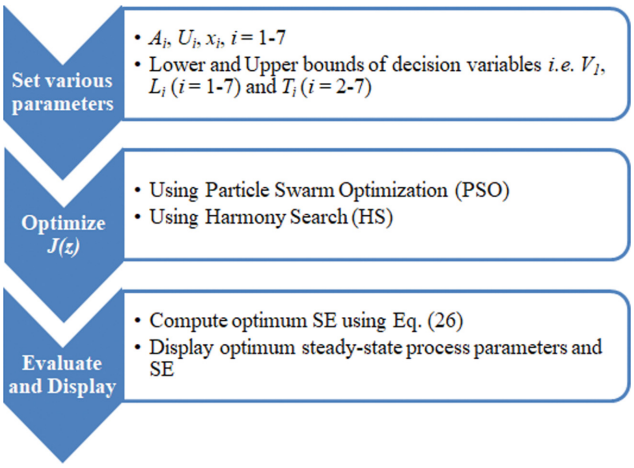

Fig. 4. Estimation of optimum steady-state process parameters and SE.

2-1. Solution of SODEs: Approximation Approach

2-1-1. Fourier Series Representation

The first step towards obtaining the solution of SODEs is the representation of the dependent parameters of the SODEs in terms of Fourier series. The SODEs (expressed by Eq. (14)-(16) and Eq. (21)-(23)) hold exit liquor concentrations (x_i, i=1-7) and temperatures of vapor emanating from each MEE effect (T_i, i=1-7) as the dependent variables. These dependent parameters are articulated in the form of partial sum of the Fourier series encompassing determinate terms of sine and cosine functions) with center t_0. Thus, x_i, (i=1-7) and T_i (i=1-7) may be expressed as given by Eq. (27)-(28).

$$x_i(t) \approx X_{i_{appx.}}(t) = a_0 + \sum_{m=1}^{NT}\left(a_m \cos\frac{m\pi(t-t_0)}{L} + b_m \sin\frac{m\pi(t-t_0)}{L}\right), \quad i=1\text{-}7 \tag{27}$$

$$T_i(t) \approx T_{i_{appx.}}(t) = c_0 + \sum_{m=1}^{NT}\left(c_m \cos\frac{m\pi(t-t_0)}{L} + d_m \sin\frac{m\pi(t-t_0)}{L}\right), \quad i=1\text{-}7 \tag{28}$$

As represented in Eq. (27)-(28), x(t) and T(t) symbolize the precise analytical solutions of SODEs. Also, X_{appx} and T_{appx} exemplify the approximate solutions of these SODEs. t_0 and t_n are the initial and final points of the solution interval such that L (which indicates the size of solution interval in compliance with a single ODE) is set to $t_n - t_0$. NT represents the number of approximation terms in the Fourier series expansion of x(t) and T(t), *i.e.*, the number of sine and cosine terms used for obtaining the approximate solutions of SODEs. The unknown coefficients in Eq. (27)-(28) (a_0, a_m, b_m, c_0, c_m and d_m), which are the coefficients of Fourier series, are computed using metaheuristic approaches (PSO and HS). The coefficients of Fourier series are considered as design variables of our optimization model for the ODEs.

2-1-2. Formulation of Objective Function

The SODEs are transformed into the optimization problem by formulating the objective function for carrying out the optimization task. The weighted residual function (WRF) is the objective function to be minimized and is expressed by Eq. (29). Here, W(t) characterizes the weight function which is anticipated to be 1 for computational simplicity. Hence, unit weight function has been preferred as conveyed by Eq. (30). R(t) denotes the residual function obtained by substituting the approximate functions (X_{appx} and T_{appx}) and their derivatives ($X'_{appx.}$ and $T'_{appx.}$) (from Eq. (27)-(28) and Eq. (31)-(32), respectively) in Eq. (33), which represents the implicit form of an ODE.

$$WRF = \int |W(t)| \times |R(t)| dt \tag{29}$$

$$W(t) = 1 \tag{30}$$

$$x_i'(t) \approx X'_{i_{appx.}}(t) = \sum_{m=1}^{NT}\left[\frac{-m\pi}{L} a_m \sin\frac{m\pi(t-t_0)}{L} + \frac{m\pi}{L} b_m \cos\frac{m\pi(t-t_0)}{L}\right], \quad i=1\text{-}7 \tag{31}$$

$$T_i'(t) \approx T_{i_{appx.}}'(t) = \sum_{m=1}^{NT}\left[\frac{-m\pi}{L}c_m \sin\frac{m\pi(t-t_0)}{L} + \frac{m\pi}{L}d_m \cos\frac{m\pi(t-t_0)}{L}\right],$$

i=1-7 (32)

$$R(t) = f(t, x_i(t), x_i'(t), T_i(t), T_i'(t)) \quad (33)$$

2-1-3. Defining the Initial Conditions

The steady-state process parameters, T_i (i=1-7), are obtained by simulation of the non-linear steady-state model of MEE using the available data. The exit liquor concentration from each MEE effect (x_i, i=1-7) and the steady-state estimates of temperatures of vapor generated at each effect of MEE (T_i, i=1-7) are treated as the initial conditions for liquor concentrations and vapor temperatures involved in SODEs. These initial conditions serve as the constraints of the articulated optimization problem.

2-1-4. Constraint Handling and Penalty Function

Moreover, to ensure that the approximate solutions satisfy the initial conditions, cautious consideration for handling constraints becomes essential. Hence, the absolute violations owing to the initial conditions are also evaluated as a methodology for handling the constraints. Subsequently, the penalty function (PF) is formed by adding the absolute values of the constraint violations. This PF is further added to the WRF by exploiting the penalty approach. In the present analysis, in order to avoid computational complexity, the penalty coefficients of 20,000 and 100 are chosen for x_i (i=1-7) and T_i (i=1-7), respectively. However, the choice of the penalty coefficients is a crucial concern. A comprehensive discussion related to the assignment of these coefficients is beyond the scope of this article. Hence, the specifications behind the selection of the penalty coefficients in the PF have not been provided.

2-1-5. Parameter Settings

In this work, the size of the solution interval L is set to 0.001 (in hour). The unknown coefficients in Eq. (27)-(28) (a_0, a_m, b_m, c_0, c_m, and d_m) are the design variables of the formulated optimization model (WRF). NT, the number of approximation terms, has been chosen to be 3. With NT=3, the total number of design variables is 98. However, owing to the dependency of approximate solutions of SODEs on the choice of number of approximation terms (NT) in the Fourier series expansion, the results may be unsatisfactory. Thus, the effect of selection of number of approximation terms on the optimization results is an eye-catching subject in the present context. Hence, for obtaining significant improvement in the simulation results, the optimization task is further performed with NT taken as 6 and 9. When NT is selected as 6 and 9, the total number of design variables becomes 182 and 266, respectively.

2-1-6. Crux Of Optimization Task

In the present analysis, the core objective of the optimization task performed by PSO and HS is to minimize the WRF by evaluation of optimum estimates of the design variables (a_0, a_m, b_m, c_0, c_m and d_m). As enunciated by Eq. (29), the objective function of the articulated optimization model (WRF) encompasses an integral term. To obtain the numerical solution of this integral, *i.e.*, the objective function (WRF), the trapezoidal method is employed as a problem-solving approach. For the proper convergence of the metaheuristic approaches, the numerical solution of the objective function must tend towards zero.

2-1-7. Solution Methodology

The problem-solving methodology employed in this work for obtaining the approximate solutions of the SODEs is exemplified in Fig. 5.

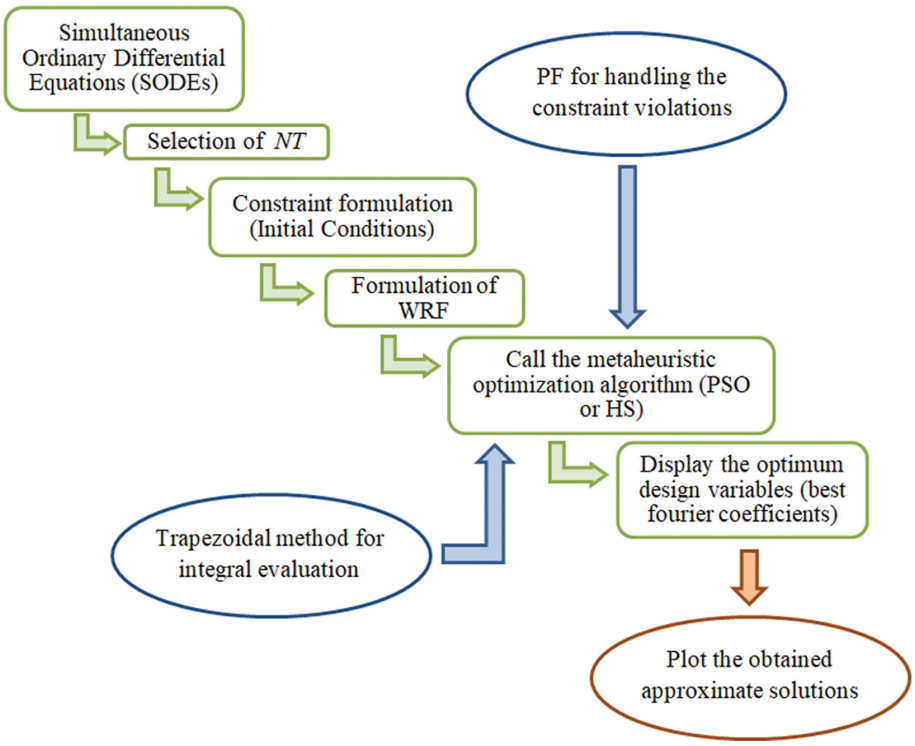

Fig. 5. Sketch of the problem-solving methodology for solving SODEs.

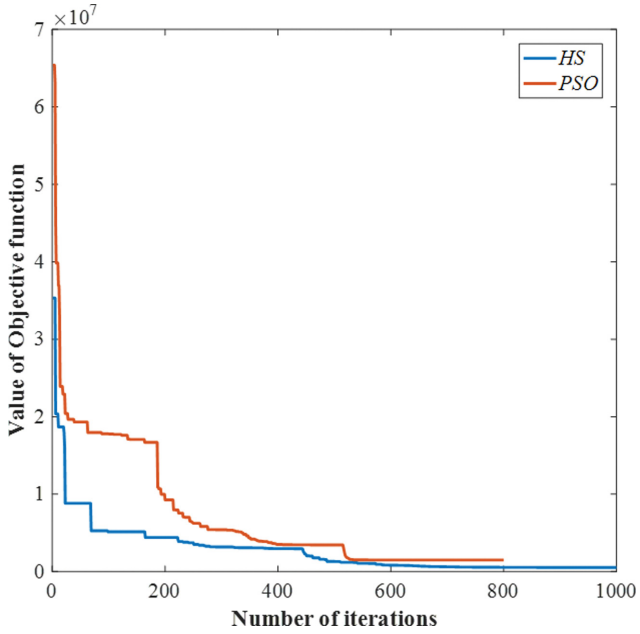

Fig. 6. Convergence curves of PSO and HS towards optimizing the objective function.

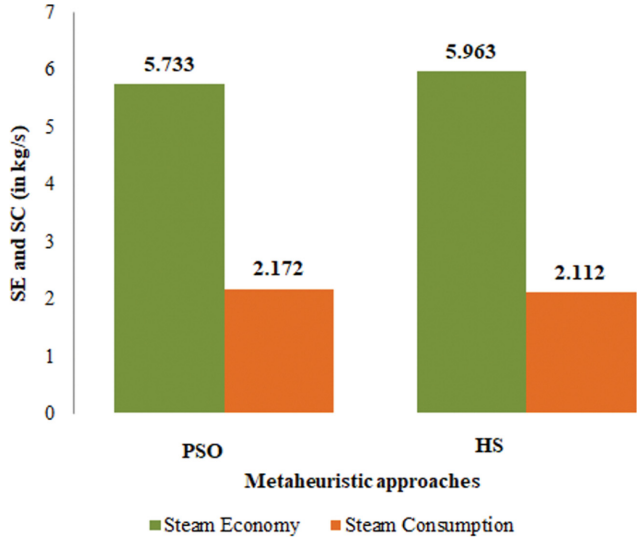

Fig. 7. Comparative analysis of PSO and HS in terms of optimum estimates of SE and SC.

OPTIMIZATION RESULTS

In the present investigation, two well-known metaheuristic techniques are used to optimize the objective function **J(z)** to acquire the steady-state process parameters comprising of L_i, T_i and V_1. For this purpose, the optimization task is carried out using 20 independent runs for each metaheuristic approach, 1000 iterations for each run. The convergence curves of PSO and HS in optimizing the formulated objective function are illustrated in Fig. 6. Evidently, both PSO and HS show satisfactory and acceptable convergence towards optimization task and finding the near-optimal solutions.

The optimum steady-state process parameters signifying the non-linear steady-state model solution have been tabulated in Table 4. In addition, for comparative assessment, steam economy (SE) and various thermo-physical parameters (λ, H and h_L or h) have also been presented in Table 4. The simulation results reveal the effectiveness of PSO and HS approaches to optimize such a complex non-linear model with the aim of hunting the optimum values of the unknown steady-state process parameters. Moreover, for comparative insight and evaluation, a relative analysis of the obtained simulation results in terms of optimum SC and SE acquired by the aforementioned metaheuristic techniques is given in Fig. 7.

As evident from Table 4 and Fig. 7, the HS algorithm yields higher

Table 4. Steady-state model solution: Optimization results of the non-linear steady-state energy model of MEE

Effect number	Metaheuristic algorithms	Steady-state process parameters					Steam economy, SE	Steam consumption (SC in kg/s) *i.e.*, V_1
		L_i (in kg/s)	T_i (in °C)	λ_i (in kJ/kg)	H_i (in kJ/kg)	h_{Li} (in kJ/kg)		
1	PSO	3.15	147	2,109.51	2,749.09	439.80		
	HS	3.01	147	2,109.51	2,749.09	439.80		
2	PSO	5.11	100	2,251.53	2,672.65	321.00		
	HS	4.85	100	2,250.86	2,673.03	321.75		
3	PSO	7.00	72.73	2,328.06	2,626.66	247.95		
	HS	6.57	73.98	2,319.01	2,632.31	259.77		
4	PSO	9.00	66.00	2,343.64	2,616.79	239.55	5.733 (PSO),	2.172 (PSO),
	HS	8.34	67.79	2,335.57	2,621.93	250.87	5.963 (HS)	2.112 (HS)
5	PSO	10.70	60.00	2,355.55	2,609.11	229.76		
	HS	10.11	61.94	2,351.79	2,611.54	235.28		
6	PSO	12.25	56.75	2,363.72	2,603.77	221.99		
	HS	11.89	58.13	2,362.39	2,604.64	223.99		
7	PSO	13.75	53.99	2,372.08	2,598.25	212.10		
	HS	13.61	54.53	2,372.23	2,598.15	211.86		

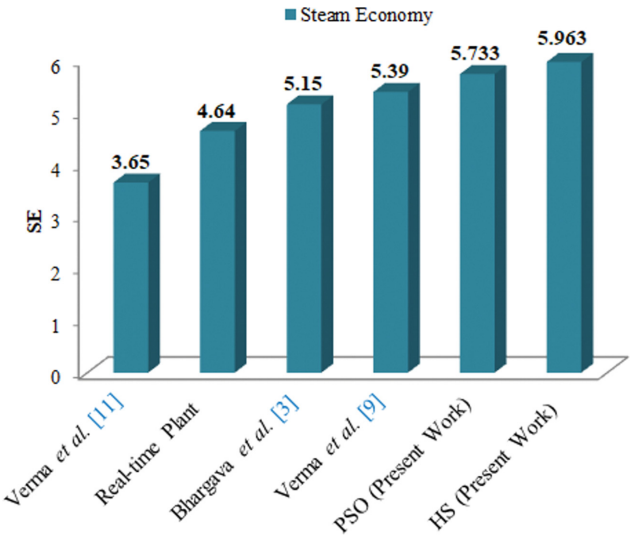

Fig. 8. Comparative sketch of SE of real-time plant, erstwhile works and present investigation.

SE (5.963) than the PSO algorithm (5.733). Moreover, a reduced SC (2.112 kg/s) is estimated by HS algorithm, which is considerably lower than that obtained by PSO algorithm (2.172 kg/s). Furthermore, a comparative assessment, as shown in Fig. 8, has been made to validate the effectiveness of PSO and HS in the computation of SE by comparing the present simulation results with other approaches in the existing literature. From Fig. 8, it can be inferred that the present results (SE) obtained by PSO and HS are significantly better as compared to the previous literature and real-time plant estimates. Consequently, PSO and HS corroborate their competence and efficiency in obtaining the solutions of complex SNLAEs in quest of the optimum unknown steady-state process parameters.

The simulation results obtained by PSO reveal that the present work gives an improved SE of 5.733 with an enhancement of ~25% relative to the real-time plant estimates. Moreover, HS offers an improvement in SE by ~29% when compared to the real-time plant SE. Thus, the present work endorses the efficacy of PSO and HS in optimizing the energy efficiency of MEE house by estimation of optimum unknown steady-state process parameters.

After obtaining the steady-state process parameters of the MEE, the approximate solutions of the SODEs involved in the dynamic model of MEE are obtained by utilizing the problem-solving methodology as discussed earlier in Fig. 5 and in sub-section 4.2. For each of the metaheuristic approaches, the maximum number of iterations has been set to 1,000 and 10 independent runs were carried out to obtain the approximate optimum solutions of the SODEs. Fig. 9 and Fig. 10 illustrate the approximate solutions of SODEs in terms of concentration and temperature of each effect of MEE house for different values of NT obtained by PSO and HS, respectively. For graphical comparative assessment, the plots of approximate solutions of SODEs obtained by PSO and HS are obtained. These plots indicate the variation in approximate solutions owing to the choice of NT in the Fourier series expansion. The optimal values of the design variables (*i.e.*, unknown coefficients of Fourier series expansion) corresponding to different values of NT demonstrating the best estimated values by PSO and HS are tabulated in Tables 5-10 (see Appendix). As evident from Fig. 9 and Fig. 10 and Tables 5-10, the time variation of liquor concentration and temperature are approximately similar as obtained by both the metaheuristic approaches (PSO and HS) in quest of the approximate solutions of the SODEs. Also, it can be observed that the increase in NT results in an oscillatory behavior of concentration and temperature at various MEE effects as seen from their corresponding plots of time variation.

Additionally, the constraint violations (deviations from the initial conditions) for different values of NT computed using PSO and HS are presented in Table 11 (see Appendix). From Table 11, it is obvious that both the metaheuristic approaches (PSO and HS) offer minimum violation of the constraints. Competitive estimates of constraint violations are reported in this work by simulation. Thus, the constraint violations are insignificant values and hence are acceptable. Moreover, the influence of NT on the accuracy of the approximate solutions is evident from the constraint violations presented in Table 11. Clearly, with increasing NT, the average error of the approximate solutions and constraint violations show noteworthy improvement. When compared to NT=3, maximum improvement in constraint violations by 99.99% and 100% is obtained for NT=6 and 9, respectively (see Table 11). Furthermore, the amount of computational time required for carrying out the simulation tasks by PSO and HS is presented in Table 12 (see Appendix). The computational time has been provided for the aforementioned system specifications (in the Introduction). Evidently, HS takes comparatively less time for accomplishing the simulations when compared to PSO.

The above discussion of the investigational results validates the performance of PSO and HS in yielding trustworthy approximate solutions of SNLAEs and SODEs. Moreover, it is evident that these metaheuristic approaches demonstrate their efficacy in handling constraint violations along with yielding optimum Fourier series coefficients to yield the optimum approximate solutions of the SODEs involved in dynamic model of MEE house. Hence, the present work demonstrates the effectiveness and proficiency of PSO and HS in solving real-time complex non-linear optimization problems.

DISCUSSION

This work presents approximate solutions of simultaneous non-linear algebraic equations and first-order simultaneous ODEs under steady and unsteady-state, respectively, of MEE. Metaheuristic approaches, PSO and HS, have been utilized to solve these energy models. The simulation results signify the successful implementation of these metaheuristic approaches towards finding the global optimal solutions of the non-linear steady-state model, thereby providing optimum SE. Moreover, a substantial enhancement in SE of the MEE house is achieved by HS over PSO for the same number of iterations. The solution of non-linear steady state model yields unknown steady-state process operating parameters which are further utilized as initial values of the dynamic model. The optimization task is carried out to solve the SODEs tangled in the dynamic model. By treating the Fourier series expansion as the fundamental approximation function, the developed SODEs are

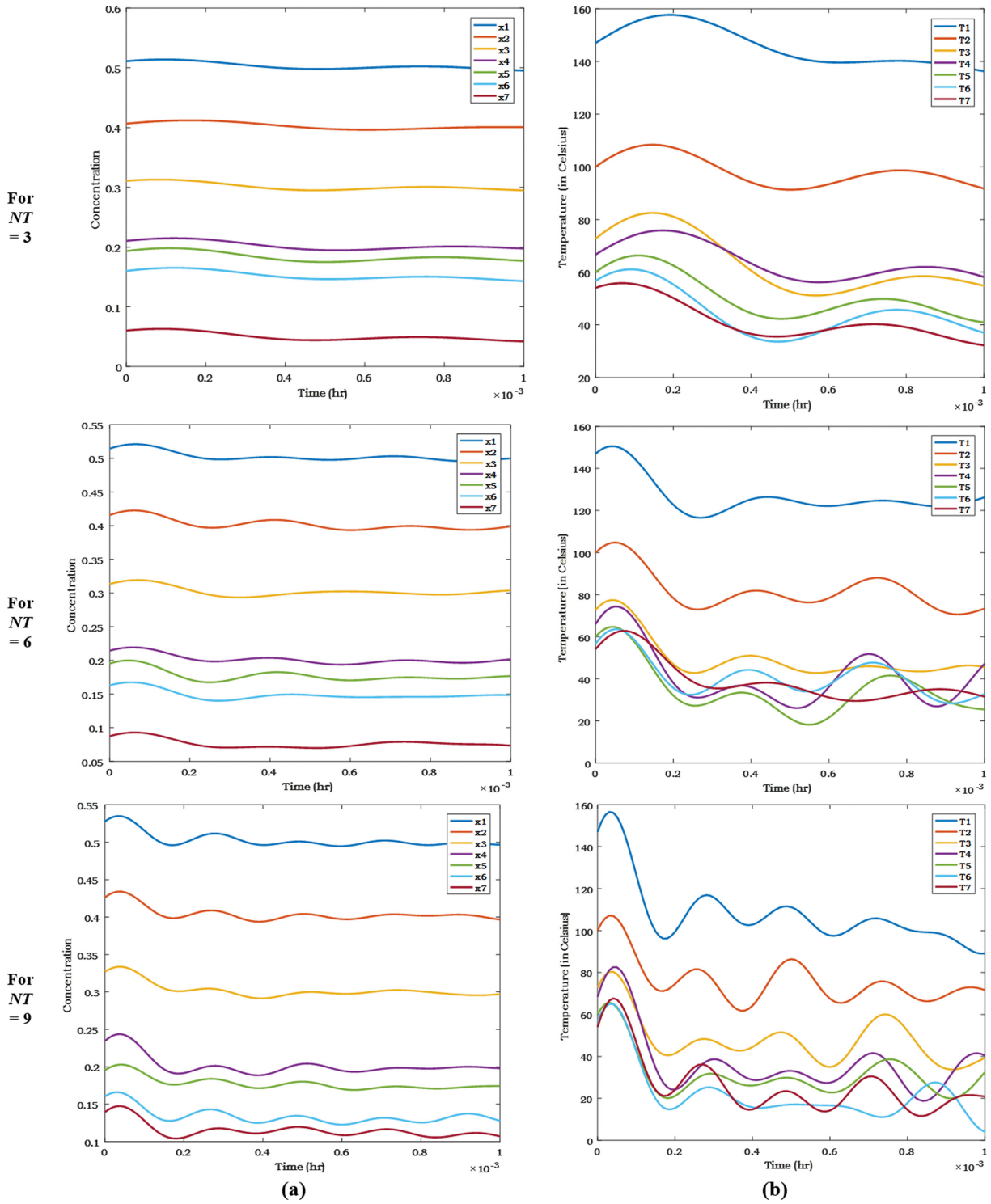

Fig. 9. Approximate solutions of SODEs by PSO: (a) Concentration of Black Liquor acquired at each effect, (b) temperature of vapor generated at each effect.

transformed into an optimization model using WRF. Further, the WRF is optimized in the hunt for the optimum values of Fourier series coefficients. Consequently, the approximate solutions of SODEs are acquired which offer information about the time variation of liquor concentrations and vapor temperatures emanating from the seven effects of the considered MEE house. Moreover, the impact of increasing number of approximation terms (Fourier coefficients) in the Fourier series expansion on the accuracy of the investigational results has been proficiently explored in this work. The present investigation, therefore, demonstrates the potential of PSO and HS in solving complex SNLAEs and SODEs involved in real time wide-ranging applications of science and engineering.

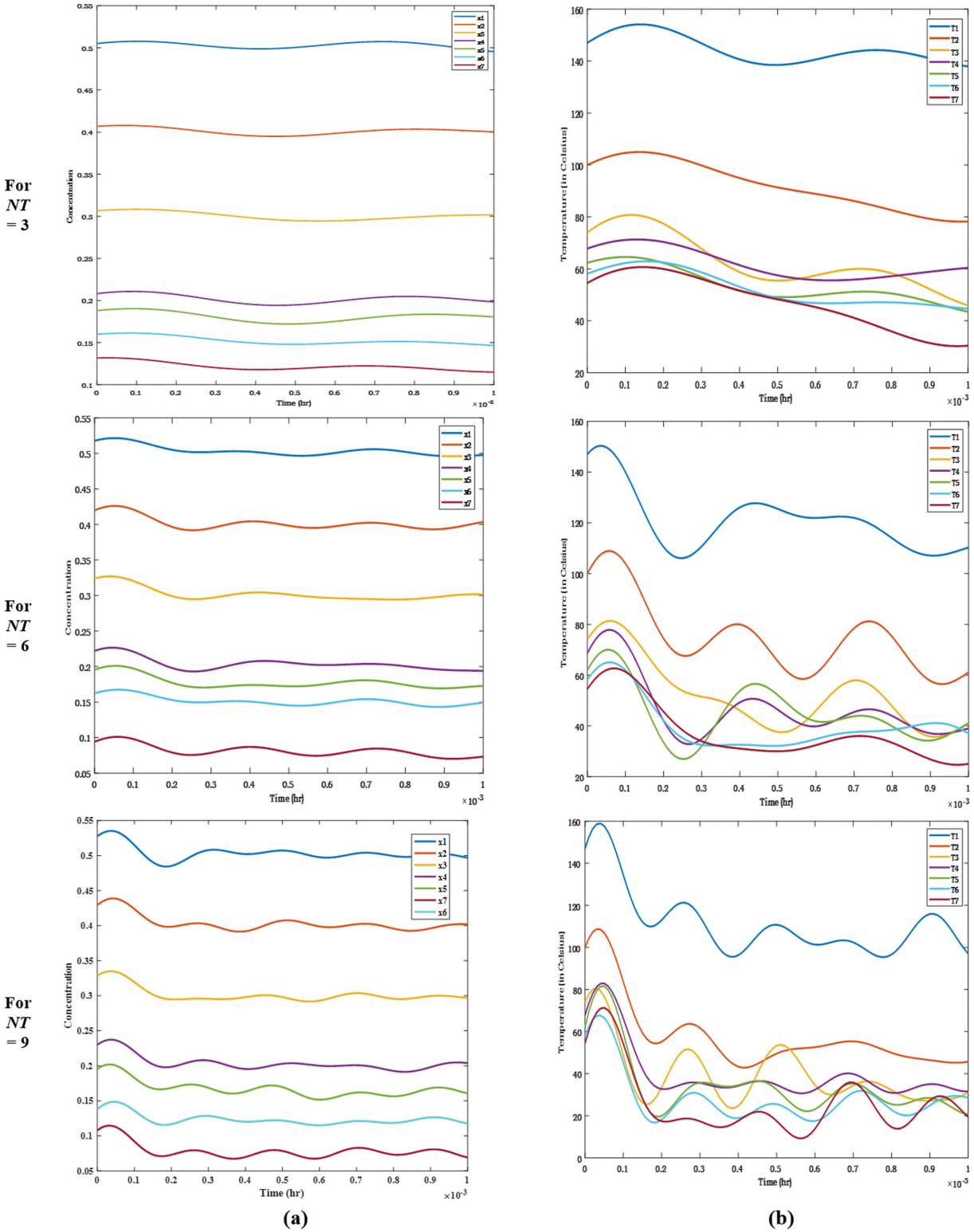

Fig. 10. Approximate Solutions of SODEs by HS: (a) Concentration of Black Liquor acquired at each effect, (b) temperature of vapor generated at each effect.

CONCLUSIONS

The time-varying (transient or open loop) behavior of liquor concentration and vapor temperature of MEE unlocks the doorway to the design of proficient controllers in an attempt to ensure energy-efficient performance of MEE. The prospect of being able to solve the steady-state and dynamic models of MEE serves as a continuous spur to future research. In our view, these results con-

stitute an excellent initial step towards controller design. Furthermore, the controller design from the dynamic model solution will ensure effective set-point tracking together with rejection of environmental disturbances and noise suppression. The design of such an advanced controller for concentration control and energy-efficient performance of MEE by utilization of dynamic model solution delivers numerous possibilities for the prospective works. Moreover, the results presented in this work can be further validated by using other metaheuristic approaches and state-of-the-art conventional techniques for solving a set of non-linear algebraic equations as well as a set of ordinary differential equations. In addition, the impact of increasing the number of approximation terms used in the Fourier series expansion on the present results can be further investigated, thereby proving to be an excellent research focus for further work.

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DECLARATION OF COMPETING INTEREST

No author associated with this paper has disclosed any potential or pertinent conflicts which may be perceived to have impending conflict with this work.

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ABBREVIATIONS AND SYMBOLS

A : effective heat transfer area [m^2]
BFFC : backward feed flow configuration
BL : black liquor
f : feed
h_L or h : enthalpy of black liquor [kJ/kg]
H : enthalpy of vapor [kJ/kg]
HEE : heptads' effect evaporator
HS : harmony search
i : effect number
J : objective function for steady-state model solution
L : liquor flow rate [kg/s]
MEE : multiple effect evaporator
N : no. of effects in the evaporator system
NT : no. of approximation terms in Fourier series expansion
ODE : ordinary differential equation
PF : penalty function
PSO : particle swarm optimization
R : residual function
SC : steam consumption [kg/s]
SE : steam economy
SNLAEs : simultaneous non-linear algebraic equations
SODEs : simultaneous ordinary differential equations
T : temperature of vapor produced [^oC]
U : overall heat transfer coefficient [$kW/m^{2o}C$]
V : vapor flow rate [kg/s]
W : weight function
WRF : weighted residual function
x : liquor concentration
z : decision variables
λ : latent heat of vaporization [kJ/kg]

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APPENDIX

Table 5. Optimum coefficients for the approximate solution of concentrations $x_i(t)$ using metaheuristic algorithms with NT=3

Function	Metaheuristic approach	Coefficients						
		a_o	a_1	b_1	a_2	b_2	a_3	b_3
$x_1(t)$	PSO	5.0000e-01	4.1571e-03	4.0704e-03	3.1381e-03	2.4761e-03	3.7301e-03	3.0166e-03
	HS	5.0000e-01	3.4258e-04	4.1076e-03	2.9740e-04	1.0267e-03	4.4489e-03	3.3605e-03
$x_2(t)$	PSO	4.0000e-01	2.8040e-03	3.8339e-03	3.5760e-03	4.1504e-03	1.9631e-04	2.2768e-03
	HS	4.0000e-01	5.7512e-04	1.0506e-03	3.5117e-03	8.3016e-04	2.7115e-03	2.2177e-03
$x_3(t)$	PSO	3.0000e-01	4.0236e-03	1.0855e-03	2.8421e-03	2.4113e-03	3.9612e-03	3.2278e-03
	HS	3.0000e-01	1.9180e-03	3.3406e-04	4.1121e-03	2.4538e-03	5.4915e-04	1.4508e-03
$x_4(t)$	PSO	2.0000e-01	4.3456e-03	2.7797e-03	3.8791e-03	3.2601e-03	2.0405e-03	4.1846e-03
	HS	2.0000e-01	9.3603e-04	1.9911e-03	3.2594e-03	1.7524e-03	3.9585e-03	3.9778e-03
$x_5(t)$	PSO	1.8000e-01	4.0261e-03	4.7391e-03	5.0000e-03	4.0651e-03	4.1375e-03	4.8385e-03
	HS	1.8000e-01	1.2462e-03	2.5751e-04	4.2375e-03	1.6375e-03	2.4788e-03	3.9051e-03
$x_6(t)$	PSO	1.5000e-01	4.7950e-03	2.5437e-03	1.4617e-03	4.0335e-03	3.7054e-03	4.8667e-03
	HS	1.5000e-01	4.2243e-03	3.2134e-03	3.2315e-03	1.0438e-03	2.3563e-03	2.0203e-03
$x_7(t)$	PSO	4.8330e-02	4.0652e-03	2.1407e-03	2.8017e-03	3.6472e-03	5.0000e-03	3.4822e-03
	HS	1.2000e-01	4.1601e-03	2.6601e-03	3.3257e-03	5.8888e-04	4.2341e-03	2.8781e-04

Table 6. Optimum coefficients for the approximate solution of temperatures $T_i(t)$ using metaheuristic algorithms with NT=3

Function	Metaheuristic approach	Coefficients						
		c_o	c_1	d_1	c_2	d_2	c_3	d_3
$T_1(t)$	PSO	1.4158e+02	4.5173e+00	5.1106e+00	7.9116e-02	5.6085e+00	8.2851e-01	4.5473e+00
	HS	1.4150e+02	1.5726e-01	3.0098e+00	9.6820e-01	5.9778e+00	4.3777e+00	5.0438e+00
$T_2(t)$	PSO	9.2796e+01	5.5690e-01	6.9737e+00	3.0810e+00	5.2575e+00	3.5703e+00	5.3767e+00
	HS	8.8421e+01	7.3848e+00	5.2676e+00	6.9696e-01	6.1260e+00	3.4970e+00	1.6407e+00
$T_3(t)$	PSO	5.9890e+01	7.0412e+00	3.4927e+00	3.9111e+00	6.7182e+00	1.8944e+00	6.8023e+00
	HS	5.7080e+01	7.9751e+00	7.1727e+00	2.8572e+00	5.2437e+00	6.0678e+00	5.9261e+00
$T_4(t)$	PSO	5.8275e+01	4.2556e+00	8.0000e+00	4.1691e+00	3.7545e+00	0	4.5545e+00
	HS	5.9144e+01	2.5084e+00	4.0440e+00	4.9497e+00	5.1736e+00	1.1845e+00	7.4390e-01
$T_5(t)$	PSO	4.7782e+01	2.4056e+00	3.0075e+00	2.6912e+00	7.3967e+00	7.1212e+00	5.6652e+00
	HS	5.2081e+01	7.0686e+00	1.3175e+00	8.2646e-01	9.3341e-01	2.3409e+00	3.4288e+00
$T_6(t)$	PSO	3.9890e+01	3.6697e+00	6.1138e+00	6.9862e+00	4.0320e+00	6.1996e+00	5.0761e+00
	HS	5.1349e+01	5.9304e+00	5.7182e-01	4.8448e-02	3.3005e+00	8.0248e-01	3.2240e+00
$T_7(t)$	PSO	3.8261e+01	5.8056e+00	4.6650e+00	4.8966e+00	2.5527e+00	5.0964e+00	2.2299e+00
	HS	4.2139e+01	7.5276e+00	7.6723e+00	3.4120e-01	7.7248e+00	4.5238e+00	1.2037e+00

Table 7. Optimum coefficients for the approximate solution of concentrations $x_i(t)$ using metaheuristic algorithms with NT=6

Coef*	$x_1(t)$		$x_2(t)$		$x_3(t)$		$x_4(t)$		$x_5(t)$		$x_6(t)$		$x_7(t)$	
	PSO	HS	PSO	HS	PSO	HS	PSO	HS	PSO	HS	PSO	HS	PSO	HS
a_o	5.0000e-01	5.0000e-01	4.0000e-01	4.0000e-01	3.0000e-01	3.0000e-01	2.0000e-01	2.0000e-01	1.7492e-01	1.7410e-01	1.4590e-01	1.5000e-01	7.3114e-02	7.7981e-02
a_1	1.9177e-03	3.6994e-03	2.6133e-03	1.6794e-03	1.2013e-03	3.9059e-03	2.7779e-03	5.0000e-03	3.1043e-03	3.5856e-03	1.5881e-03	2.5265e-03	1.7921e-03	3.7815e-03
b_1	3.0061e-03	4.5709e-03	2.3120e-03	1.5810e-03	2.2798e-03	2.1304e-03	1.7225e-03	4.4825e-03	2.3665e-03	3.6899e-03	3.1642e-03	9.5894e-04	3.3006e-03	4.1491e-03
a_2	2.8091e-03	3.5005e-03	1.8916e-03	3.6130e-03	3.4326e-03	4.6983e-03	3.7812e-03	1.0587e-03	2.5173e-03	2.8210e-03	3.5140e-03	1.1638e-03	3.0216e-03	6.3251e-04
b_2	3.5810e-03	1.6218e-03	5.0000e-03	3.1617e-03	2.3407e-03	3.9897e-03	3.0219e-03	2.6158e-04	1.9976e-03	1.7980e-03	2.1574e-03	2.5611e-03	8.9268e-04	2.2229e-03
a_3	3.4055e-03	2.3375e-03	1.6867e-03	2.5817e-03	3.5137e-03	4.2404e-03	1.4018e-03	4.0693e-03	2.1590e-03	5.0000e-03	3.5147e-03	2.5220e-03	4.3800e-03	2.5573e-03
b_3	2.6073e-03	3.4516e-03	2.3571e-03	1.3244e-03	2.9872e-03	6.1695e-04	1.7508e-03	1.8968e-03	2.5698e-03	4.4767e-03	2.1263e-03	3.3810e-03	4.8252e-03	3.0513e-03
a_4	1.4336e-03	1.6963e-03	3.3292e-03	3.0467e-03	4.2852e-03	3.8852e-03	1.7882e-03	3.2941e-03	4.7861e-03	3.3115e-03	3.5959e-03	1.2105e-03	1.1750e-03	1.3290e-03
b_4	3.2218e-03	4.1666e-03	2.9083e-03	4.5280e-03	3.6693e-03	8.9875e-04	1.9940e-03	3.4272e-03	1.9071e-03	4.3868e-03	1.7329e-03	3.2733e-03	9.5515e-04	3.2068e-03
a_5	1.9815e-03	3.9537e-03	4.0679e-03	3.8738e-03	1.3900e-04	3.0819e-03	2.2206e-03	5.0000e-03	4.0085e-03	2.7521e-03	2.1679e-03	1.4888e-03	8.1913e-04	4.1487e-03
b_5	2.6867e-03	5.5837e-04	2.3425e-03	3.5604e-03	1.7971e-03	4.8357e-03	1.9049e-03	4.4736e-03	3.6065e-03	2.2978e-03	4.0452e-03	1.0857e-03	3.3475e-03	2.7168e-03
a_6	2.9931e-03	2.5648e-03	2.1791e-03	5.0000e-03	9.1984e-04	4.1221e-03	2.4337e-03	3.6332e-03	3.7386e-03	3.8293e-03	2.5702e-03	3.3674e-03	3.1206e-03	3.6659e-03
b_6	2.7599e-03	1.4614e-03	4.1046e-03	4.4366e-03	1.4377e-03	1.3706e-03	3.2543e-03	2.3222e-03	3.5025e-03	2.5084e-03	1.8957e-03	2.8132e-03	2.1326e-03	4.4885e-03

*Coefficient

Table 8. Optimum coefficients for the approximate solution of temperatures $T_i(t)$ using metaheuristic algorithms with NT=6

Coef*	$T_1(t)$		$T_2(t)$		$T_3(t)$		$T_4(t)$		$T_5(t)$		$T_6(t)$		$T_7(t)$	
	PSO	HS	PSO	HS	PSO	HS	PSO	HS	PSO	HS	PSO	HS	PSO	HS
a_o	1.2210e+02	1.1675e+02	7.8168e+01	7.1443e+01	4.5531e+01	4.6162e+01	3.8516e+01	4.0888e+01	3.1082e+01	4.1491e+01	3.6701e+01	3.6391e+01	3.5855e+01	3.3941e+01
a_1	1.9261e+00	5.2431e+00	3.0175e+00	7.3770e+00	3.9806e+00	7.1770e+00	7.6142e-01	4.9948e+00	4.2383e+00	2.4551e+00	1.8065e+00	1.6570e+00	4.5701e+00	4.0045e+00
b_1	6.3233e+00	5.6276e+00	4.7851e+00	3.0273e+00	4.1828e+00	7.2485e+00	2.9040e+00	7.8584e+00	3.1462e+00	4.0934e+00	5.1995e+00	3.1327e+00	2.5902e+00	3.0181e+00
a_2	7.5225e+00	1.5785e+00	3.0620e+00	3.0766e+00	3.3304e+00	6.5427e+00	7.9697e+00	6.7678e+00	6.6593e+00	3.3134e-01	1.7044e+00	5.2129e+00	3.1307e+00	3.7492e+00
b_2	2.2451e+00	1.3633e+00	1.8056e+00	4.6106e+00	4.5524e+00	3.6369e+00	4.3406e+00	2.1538e+00	2.4798e+00	1.9193e+00	3.1120e+00	4.0064e+00	5.8441e+00	6.0988e+00
a_3	2.7191e+00	5.7709e+00	6.0511e+00	7.5300e+00	5.4001e+00	5.5446e+00	6.3101e+00	4.9237e+00	6.7335e+00	3.3516e+00	4.2375e+00	5.6041e+00	3.2198e+00	5.2111e+00
b_3	7.9596e-01	5.4651e-10	3.0987e+00	5.7522e+00	4.8576e+00	5.2908e+00	3.1844e+00	3.8605e-01	5.1695e+00	4.9547e-01	2.5698e+00	5.5756e+00	4.4213e+00	4.5487e+00
a_4	5.1254e+00	5.8174e+00	2.6613e+00	1.8523e+00	4.6905e+00	1.0068e-01	2.5800e+00	2.2165e+00	1.4190e-01	5.8696e+00	6.6707e-01	1.5394e+00	2.0226e+00	2.3313e-01
b_4	5.5072e+00	5.3980e+00	3.3858e+00	1.4384e+00	1.6585e+00	6.7351e+00	6.9804e+00	4.2610e+00	1.3349e+00	3.9356e+00	5.4226e+00	3.1463e+00	3.2410e+00	7.1473e+00
a_5	5.7166e+00	7.3336e+00	4.1919e+00	4.5238e+00	4.2797e+00	3.8721e+00	2.3780e+00	4.9241e+00	6.3409e+00	4.6387e+00	5.9268e+00	3.2981e+00	3.4839e+00	5.5041e+00
b_5	1.6097e+00	3.3932e+00	1.1817e+00	2.0704e+00	4.5844e+00	5.3240e-01	1.1420e+00	6.5027e+00	3.4719e+00	5.2633e+00	3.2578e+00	6.3257e+00	5.5117e+00	3.0048e+00
a_6	1.8889e+00	4.5177e+00	2.7284e+00	4.1972e+00	5.5173e+00	4.5891e+00	7.4808e+00	3.9619e+00	4.8312e+00	3.8080e+00	5.6236e+00	4.4395e+00	1.7709e+00	1.8889e+00
b_6	1.4809e+00	2.2304e+00	3.8057e+00	8.0000e+00	1.7378e+00	2.8081e+00	6.8798e+00	5.2858e+00	3.3748e+00	6.2248e+00	3.9968e+00	2.5085e-01	6.1521e-01	3.6637e-10

*Coefficient

Table 9. Optimum coefficients for the approximate solution of concentrations $x_i(t)$ using metaheuristic algorithms with NT=9

Coef*	$x_1(t)$		$x_2(t)$		$x_3(t)$		$x_4(t)$		$x_5(t)$		$x_6(t)$		$x_7(t)$	
	PSO	HS	PSO	HS	PSO	HS	PSO	HS	PSO	HS	PSO	HS	PSO	HS
a_o	5.0000e-01	5.0000e-01	4.0000e-01	4.0000e-01	3.0000e-01	2.9946e-01	2.0000e-01	2.0000e-01	1.7423e-01	1.6432e-01	1.3153e-01	7.5212e-02	1.1161e-01	1.2000e-01
a_1	2.8583e-03	1.8075e-03	1.5172e-03	3.8524e-03	2.8756e-03	3.6103e-03	2.7945e-03	4.7755e-03	3.4853e-03	5.0000e-03	3.4517e-03	2.0880e-03	3.1913e-03	1.1733e-03
b_1	3.3251e-03	4.1550e-03	4.2416e-03	3.5754e-03	1.6513e-03	1.0498e-03	0	3.6207e-03	3.4374e-03	2.1818e-03	1.2907e-03	2.9980e-03	3.0885e-03	3.2097e-03
a_2	4.5886e-03	2.5102e-03	3.7369e-03	4.5380e-03	3.7355e-03	4.7960e-03	4.7944e-03	3.8731e-03	2.5335e-03	4.7417e-03	2.9714e-03	4.6093e-03	2.9234e-04	2.6899e-03
b_2	3.7916e-03	1.9103e-03	3.0574e-03	2.7768e-03	3.0881e-03	1.0951e-03	2.6476e-03	2.2960e-03	3.5049e-03	4.5256e-03	3.2141e-03	1.7716e-03	2.9149e-03	2.6780e-03
a_3	2.0500e-03	3.7089e-03	3.0889e-03	5.0000e-03	4.5885e-03	3.6956e-03	5.0000e-03	2.1843e-03	1.3211e-03	1.9765e-03	1.9607e-03	3.7666e-03	4.5366e-03	1.0646e-04
b_3	2.0471e-03	6.9021e-04	3.6068e-03	2.3890e-03	4.2268e-03	1.6331e-03	2.3512e-03	2.6102e-03	3.0105e-03	2.7979e-03	4.4996e-03	3.7510e-03	2.3480e-03	2.8144e-03
a_4	2.3008e-03	4.7258e-03	1.7531e-03	5.0000e-03	2.6425e-03	2.2552e-03	4.4559e-03	3.9586e-03	2.7975e-03	3.3545e-03	3.8808e-03	4.1068e-03	2.3300e-03	1.2987e-03
b_4	2.9464e-03	4.7492e-04	3.3840e-03	3.7095e-03	4.4768e-03	3.1713e-03	3.4091e-03	3.9470e-03	2.3170e-03	1.3554e-03	1.4182e-03	3.7798e-03	1.3490e-03	1.8746e-03
a_5	2.8636e-03	5.0000e-03	3.4013e-03	9.5480e-04	3.6517e-03	3.6808e-03	3.8752e-03	1.3544e-03	8.4333e-04	2.4265e-03	2.2339e-03	4.3983e-03	3.1458e-03	1.9458e-03
b_5	3.5114e-04	1.3902e-03	3.1670e-03	2.4850e-03	2.3186e-03	4.0987e-03	4.3343e-03	3.3538e-03	2.6146e-03	3.6856e-03	1.5737e-03	1.2757e-03	4.7479e-03	3.1306e-03
a_6	2.3491e-03	3.0832e-03	1.7613e-03	1.6106e-03	2.8790e-03	3.9268e-03	2.8364e-03	5.0000e-03	2.0543e-03	1.2680e-03	2.4459e-03	1.2565e-03	4.4661e-03	1.9933e-03
b_6	3.2901e-03	3.3229e-03	2.1047e-03	3.5066e-03	1.0794e-03	4.2578e-03	2.6897e-03	2.8300e-03	3.1467e-03	3.8699e-03	2.2279e-03	3.8492e-03	3.3827e-03	4.5006e-03
a_7	4.0963e-03	3.8880e-03	3.5610e-03	2.3015e-03	1.1048e-03	1.6237e-03	3.3591e-03	3.8822e-03	2.6531e-03	3.4768e-03	4.4139e-03	4.0883e-03	2.7011e-03	4.3400e-03
b_7	2.7491e-03	3.2987e-03	2.8170e-03	2.9406e-03	7.3404e-04	2.6175e-03	2.5840e-03	3.0231e-03	1.0408e-03	2.9770e-03	2.1040e-03	3.1868e-03	3.1866e-03	3.7799e-03
a_8	3.0428e-03	1.9139e-03	4.2697e-03	4.5015e-03	2.8647e-03	2.2393e-03	4.1749e-03	4.0567e-03	3.2546e-03	4.2871e-03	3.3315e-03	3.2779e-03	4.5161e-03	1.7218e-03
b_8	2.8794e-03	2.2745e-03	2.8689e-03	2.7734e-03	4.0776e-03	1.0675e-03	4.6481e-03	1.4155e-03	2.7081e-03	4.4122e-04	1.2492e-03	7.1504e-04	2.1177e-03	1.8029e-03
a_9	3.7416e-03	1.0205e-03	3.2594e-03	1.6101e-03	2.7389e-03	3.2075e-03	3.2474e-03	9.5161e-04	2.2304e-03	4.5496e-03	4.2337e-03	5.0000e-03	2.6066e-03	2.6575e-03
b_9	3.5712e-03	4.6313e-03	2.6759e-03	3.1974e-03	1.5334e-03	1.9061e-03	3.4695e-03	2.4054e-03	2.4589e-03	3.4887e-03	4.4273e-03	4.4005e-03	3.3987e-03	3.6426e-03

*Coefficient

Table 10. Optimum coefficients for the approximate solution of temperatures $T_i(t)$ using metaheuristic algorithms with NT=9

Coef*	$T_1(t)$		$T_2(t)$		$T_3(t)$		$T_4(t)$		$T_5(t)$		$T_6(t)$		$T_7(t)$	
	PSO	HS	PSO	HS	PSO	HS	PSO	HS	PSO	HS	PSO	HS	PSO	HS
c_o	1.0293e+02	1.0874e+02	7.3210e+01	5.3195e+01	4.2618e+01	3.6421e+01	3.2511e+01	3.2641e+01	2.7545e+01	2.9854e+01	1.5720e+01	2.5387e+01	2.1079e+01	2.0821e+01
c_1	4.4246e+00	5.5118e+00	2.3991e+00	6.8321e+00	2.3434e+00	5.9183e+00	2.9258e+00	1.7439e+00	1.6256e-01	3.3334e+00	3.8499e+00	2.5233e+00	3.6952e+00	3.2787e+00
d_1	5.2485e+00	0	1.6172e+00	3.9382e+00	6.8447e+00	3.1241e+00	3.4629e+00	7.4564e+00	4.1843e+00	2.7929e+00	7.2083e+00	7.4139e-01	4.3773e+00	3.9286e+00
c_2	2.1619e+00	4.2248e+00	5.4638e-01	6.5967e+00	3.6234e+00	1.8825e+00	4.0985e+00	5.5208e+00	3.9509e+00	4.0973e-01	6.3848e+00	1.5995e+00	3.0550e+00	6.4073e+00
d_2	7.5610e+00	5.5476e+00	3.3443e+00	5.2111e+00	2.4220e+00	9.9470e-01	4.8235e+00	6.0776e+00	2.4909e+00	4.2937e+00	5.1406e+00	1.5924e+00	5.5342e+00	1.5831e+00
c_3	6.8926e+00	3.1218e+00	2.3842e+00	7.8972e+00	4.4961e+00	1.0330e+00	5.4332e+00	6.0197e+00	2.9022e+00	3.0152e+00	5.0146e+00	2.2742e+00	3.8479e+00	5.4677e+00
d_3	3.7705e+00	4.8726e+00	3.2706e+00	4.8348e+00	2.9967e+00	7.7544e-01	5.4838e+00	5.1221e+00	3.7671e+00	4.5872e+00	4.4221e+00	7.4883e+00	7.0593e+00	5.0328e+00
c_4	3.6294e+00	2.3443e+00	3.4090e+00	3.7739e+00	1.7528e+00	5.7149e+00	6.6395e+00	4.3451e+00	4.5023e+00	2.2819e+00	3.7237e+00	6.0114e+00	4.5690e+00	2.1595e+00
d_4	3.2166e+00	5.4622e+00	5.3801e+00	4.9684e+00	3.5572e+00	6.8954e+00	3.9376e+00	4.3952e+00	3.8488e+00	5.0833e+00	3.3958e+00	3.9835e+00	5.1309e+00	4.4999e+00
c_5	6.6303e+00	2.9753e+00	3.6530e+00	3.1575e+00	5.7458e+00	4.8950e+00	1.5714e+00	2.1295e+00	3.6025e+00	6.2755e+00	7.1561e+00	3.1392e+00	2.5912e+00	2.0347e+00
d_5	4.0970e+00	6.8686e+00	4.4590e+00	1.2374e+00	6.1044e-01	6.7611e-01	2.4118e+00	4.7435e+00	7.2239e-01	6.2568e+00	4.7699e+00	2.5922e+00	5.7958e-01	6.2980e+00
c_6	6.1076e+00	1.7707e+00	1.3225e+00	2.9264e+00	1.2438e+00	1.2359e+00	6.4170e+00	3.9935e+00	5.1819e+00	4.4981e+00	3.1580e+00	6.3471e+00	3.7763e+00	5.9799e+00
d_6	4.5413e+00	2.4485e+00	1.7826e+00	4.2800e+00	6.0984e+00	8.7990e-01	7.1954e+00	7.4458e+00	8.0000e+00	6.5114e+00	9.7606e-01	5.3884e+00	3.3270e+00	7.1389e+00
c_7	5.4483e+00	6.8052e+00	2.8925e+00	5.8536e+00	1.1991e+00	4.2133e+00	3.3056e+00	3.3736e+00	4.8543e+00	1.2185e+00	5.4476e+00	3.7137e+00	5.2264e-01	9.2801e-01
d_7	6.3523e-01	5.7093e+00	1.7924e+00	5.7403e+00	2.5503e+00	6.8527e-01	5.2584e+00	5.0896e+00	0	7.3120e+00	3.5575e+00	1.9024e+00	3.8461e+00	5.0429e+00
c_8	3.2181e+00	4.9677e+00	7.3406e+00	6.3740e+00	6.7782e+00	7.2827e+00	4.7853e+00	3.1728e+00	5.0613e+00	4.3691e+00	1.4103e+00	4.0348e+00	4.9777e+00	1.5963e+00
d_8	4.8688e+00	2.6861e+00	2.0688e+00	3.7805e+00	3.1460e+00	6.7639e+00	1.5015e+00	3.5726e+00	3.0817e+00	7.0874e+00	4.3889e+00	2.7170e+00	5.5966e+00	2.0352e+00
c_9	5.5618e+00	6.5352e+00	2.8422e+00	3.3930e+00	2.9286e+00	5.3830e+00	7.4475e-01	4.8499e+00	2.2362e+00	6.9383e+00	4.8806e+00	3.1004e+00	5.8760e+00	5.8570e+00
d_9	4.7212e+00	4.4505e+00	3.2559e+00	1.1402e+00	1.7238e+00	4.6327e+00	4.5907e+00	2.0559e+00	6.1084e-01	3.7033e+00	1.7827e+00	4.1011e+00	5.2195e+00	5.1599e+00

*Coefficient

Table 11. Comparative analysis of the constraint violations (initial condition violations) using PSO and HS

Function	Metaheuristic approach	Deviation values from initial conditions (e)				
		for NT=3	for NT=6	Error improvement using NT=6 w.r.t NT=3 $\%e=\frac{e_{NT=3}-e_{NT=6}}{e_{NT=3}}\times 100$	for NT=9	Error improvement using NT=9 w.r.t NT=3 $\%e=\frac{e_{NT=3}-e_{NT=9}}{e_{NT=3}}\times 100$
$x_1(t)$	PSO	1.7575e-02	1.4060e-02	20	7.0884e-04	95.96
	HS	2.3511e-02	1.0848e-02	53.85	9.4205e-04	95.99
$x_2(t)$	PSO	2.5524e-02	1.6332e-02	36.01	5.7512e-03	77.46
	HS	2.5302e-02	1.2305e-02	51.36	2.7310e-03	89.20
$x_3(t)$	PSO	1.7673e-02	1.5008e-02	15.07	1.4187e-03	91.97
	HS	2.1921e-02	4.5662e-03	79.16	5.9286e-14	100
$x_4(t)$	PSO	3.6235e-02	3.2096e-02	11.42	1.1962e-02	66.98
	HS	3.8346e-02	2.4445e-02	36.25	1.6464e-02	57.06
$x_5(t)$	PSO	2.2363e-03	1.6282e-04	92.71	4.3033e-07	99.98
	HS	7.4375e-03	4.3390e-06	99.94	3.8411e-13	99.99
$x_6(t)$	PSO	2.5379e-03	3.4838e-04	86.27	2.0419e-03	19.54
	HS	2.6878e-03	2.2110e-04	91.77	5.4696e-02	−1,934
$x_7(t)$	PSO	7.9203e-02	5.1978e-02	34.37	1.2642e-06	99.99
	HS	7.6801e-03	4.5304e-02	−489	1.4733e-03	80.81
$T_1(t)$	PSO	1.1369e-13	1.4595e-08	−12e+06	6.1621e-05	−5e+10
	HS	6.6870e-07	6.8349e-03	−10e+05	1.0168e-04	−15e+03
$T_2(t)$	PSO	4.0388e-03	1.1948e-01	−2858	9.9667e-05	97.53
	HS	7.3620e-05	8.0362e-11	99.99	4.6140e-04	−526
$T_3(t)$	PSO	6.6078e-03	2.8034e-10	99.99	4.3401e-04	93.43
	HS	5.7769e-05	8.0399e-03	−13e+03	6.5809e-05	−13.91
$T_4(t)$	PSO	6.9951e-01	4.3706e-03	99.37	2.4326e+00	−247
	HS	3.3362e-03	8.8673e-01	−26e+03	1.4792e-05	99.55
$T_5(t)$	PSO	7.2459e-05	2.7466e-02	−37e+03	9.7422e-04	−1,244
	HS	3.7701e-01	5.5713e-03	98.52	2.5336e-01	32.79
$T_6(t)$	PSO	4.1998e-03	8.3434e-02	−1886	4.2443e-03	−1.05
	HS	2.2890e-05	1.2068e-02	−52e+03	9.5118e-04	−4,055
$T_7(t)$	PSO	6.9638e-02	6.2834e-02	9.77	2.1316e-14	100
	HS	1.6358e-03	1.9526e-03	−19.36	2.0734e-04	87.54

Table 12. Comparison of computational time for simulation

Metaheuristic approach	Computational time (in seconds)	
	Steady-state model solution	Dynamic model solution
PSO	20.34	543.33
HS	15.73	491.11