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A sampling-based stochastic optimization for a boiler process in a pulp industry

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Abstract−Our Aim was to find the stochastic optimal solution for a boiler process in order to maximize steam generation while complying with pollutant emissions regulations. For a simulation base-model, support vector regression (SVR) is employed to present a general discrete-time dynamical system of a boiler, and a stochastic optimization was performed to take inherent process uncertainties into account. To generate a stochastic optimization problem, sample average approximation (SAA) based on Monte-Carlo sampling was introduced due to the properties of a SVR. Moreover, a gradient free-based particle swarm optimization (PSO) technique was applied to find the optimal parameters of SVR and investigate the stochastic optimal solution for the boiler process. The results show that the stochastic optimal solution provides almost 2% more steam generation under uncertainties: this indicates that the stochastic optimal solution provides more realistic results compared to the deterministic approach. The proposed methodology can be applied straightforwardly to black box models, or when the use of gradient-based optimization solvers is restricted.

Keywords: Stochastic Optimization, Support Vector Regression, Particle Swarm Optimization, Chemical Recovery Boiler, Pulp Mill

INTRODUCTION

Ideally, mimicking real processes requires rigorous mathematical models. These are based on the first principle equations of chemical engineering, such as mass and energy balances, reaction kinetics, and so on. A satisfactory process model in hand can be used for optimization purposes. Hence, an optimal solution can be investigated for the set of designed variables. The general bottleneck is the unavailability of sufficient process knowledge because of the lack of a minimum understanding relative to the physicochemical phenomena. In addition, the general nonlinear behavior triggers trouble during process modelling and simulation. Even with knowledge availability, model development can be time-consuming and expensive.

This scenario is both the case for boilers in general and more specifically for chemical recovery boilers (as investigated in the present work) in pulp mills. One challenge with such equipment concerns emission modelling, for which pure mathematical models are not available [1,2]. In chemical recovery boilers, there are many ambiguous physicochemical phenomena, and parameter values such as fuel and air properties vary over time. Moreover, the combustion process itself is not stable generating fluctuations in response variables even when constant operation is assumed e.g. variable CO emission. Further, the char bed may show from hours to days dynamicity, where fuel residue, i.e., partially burned black liquor, is deposited (stock grows) and also consumed (stock decreases) during boiler operation.

Ghaffari and Romagnoli [3], Blasiak et al. [4], and Leiviska [5] present examples of the static and dynamic modelling of Kraft recovery boilers based on mass and energy balances. This approach always makes assumptions regarding chemical reduction efficiency, smelt temperature, boiler efficiency and excess oxygen in the stack, to mention a few. In addition, they are generally only valid for a particular case, and model parameters such as the reaction kinetics should be re-estimated for different mills. Due to this complexity, studies have generally focused on specific boiler subsystems. For instance, Perez et al. [6] presented a CFD model for ash deposition and fouling growth for a specific example of ash deposits in the boiler bank of a particular boiler. Another example is given in Wallinder et al. [7], which aimed to recognize constant changes in the process stream compositions in pulp mills based on a process simulator that is based on a dynamic model. The authors highlight the need for knowledge of the residence time between operations in the mill, which in practice is known to be a varying parameter.

To tackle the difficulties regarding process modeling, data-driven methods have been widely used as an alternative, since data availability is no more a restriction in at least medium and large process industries worldwide. In this direction, Maakala et al. [8] combined simulated annealing, local polynomial regression, and computational fluid dynamics for geometry optimization of the superheaters in the convective heat transfer section of a chemical recovery boiler. To mention some general applications in boilers, Safdarnejad et al. [9] employed a dynamic data-driven approach based on a recurrent neural network to estimate NOx and CO emissions in a utility boiler simultaneously. This approach was justified by its strong

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nonlinear characteristic. The model was used to predict into the future for optimization purposes. Pan et al. [10] highlighted the frequent variable loads in boilers. Given this, the authors used genetic algorithms to optimize the performance of the combustion control system of an industrial boiler to reach an optimal combustion interval. Wang et al. [11] applied a Gaussian process to optimize the combustion process of a tangentially fired boiler with the aim of reducing NOx emissions. They also adopted genetic algorithms for the optimization of the model hyper-parameters. The authors justified the adoption of a data-driven approach due to the complex mechanism of boiler combustion. Song et al. [12] obtained the combustion description of a utility boiler using general regression neural network in conjunction with Gaussian adaptive resonance theory. This predictive model was then used to optimize the process using the artificial bee colony algorithm, for which the objective function considered the coal consumption, NOx emissions and the recycling potential of fly ash. Zhou et al. [13] identified a predictive model for NOx emissions in a power boiler using support vector regression (SVR), for which ant colony optimization was used for the estimation of the model parameters.

In this study, the SVR-based empirical model, which has many attractive advantages for nonlinear modeling, was employed for process modeling. SVR has been shown to be one of the most effective tools for handling nonlinear information [14]. In the pulp industry field, Vincent el al. [15] employed SVR to make regression models and detect abnormal behavior. This study proved the efficacy of SVR for modeling a reboiler process.

Traditional optimization approaches aim to find the optimal solution through a mathematical model, an objective function and a set of constraints. However, in the absence of such a description, derivatives are not available and gradient-based optimization cannot be adopted to find the optimal conditions. This study adopts a sample-based approach, in which derivatives are not necessary. Furthermore, realistic processes are inherently stochastic. Thus, a deterministic approach may not be sufficient to guarantee the best solutions under uncertainty, and stochastic optimization approaches yield more realistic solutions [16,17]. Stochastic optimization techniques have been widely studied in the fields of process optimization and computer-aided molecular design [16-22]. It is generally assumed that random variables follow a specific probability function and overall complex and highly nonlinear models are then decomposed into several linear models. Throughout the convexity of such linear models, it is easier to find the stochastic solutions. An alternative is the sample-based approach, where stochastic optimal solutions make the average expectation of the objective function given scenarios minimize or maximize. These scenarios are generated through combinations of samples of random variables. Stochastic optimal solutions of a sample-based approach show the best performance in terms of averages on the objective function.

Specifically, the present work employs SVR to model a boiler process and proposes a stochastic optimization procedure. Particle swarm optimization (PSO), which is gradient-free optimization, is introduced to optimize both the SVR parameter set and the stochastic solutions. First, a set of SVR models is trained to construct rigorous models to mimic a boiler process. Their optimal parameters are determined via PSO. In sequence, scenarios are generated by sample combinations of random variables and a stochastic optimization problem using a sample-based approach is generated. PSO is then employed to find the best stochastic solution. The case study refers to a chemical recovery boiler belonging to a pulp mill in Brazil.

The paper is organized as follows. In Sections 2 and 3, the basic model of the case study and its optimization problem are introduced. Section 4 explains how to generate the stochastic problem, and Section 5 describes the PSO technique. In Section 6, the objective function is defined, and Section 7 discusses the results of the stochastic model, which are compared with the results of the deterministic solution. Finally, Section 8 gives concluding remarks.

A RECOVERY BOILER

The case study refers to a chemical recovery boiler of a Kraft pulp mill in Brazil. Its main role is the recovery of specific sodium-based compounds for reuse as cooking chemicals in the cooking stage of the wood chips, from which the cellulose pulp is obtained for papermaking. As power boilers, it also produces high-pressure steam for electric power generation and heat exchange operations in the mill. Its fuel is an aqueous solution composed of organic and inorganic compounds called residual liquor, which is the byproduct of the cooking stage. Before being burned in the boiler, it is concentrated from about 20% to above 60% in mass by partially evaporating its water content. This operation occurs in an evaporation station. The last recovery step for the cooking chemicals occurs in a causticizing stage. Fig. 1 shows the main stages of Kraft pulp mills with an emphasis on the boiler [22-24].

This boiler has two main sections: furnace and heat recovery. This section starts with radiative heat transfer controlling the flue gas cooling in the superheaters, changing to convective heat transfer in the remaining heat exchangers, namely, boiler bank and economizers. The first is very similar to a chemical reactor, where the combustion process and the recovery of the inorganic compounds occur. Combustion air is injected at three levels: primary, secondary, and tertiary. In the latter, fresh water at approximately 120 °C is transformed to high-pressure steam around 480 °C and 6.5 MPa. This is achieved with the use of the released hot fuel gases in counter-current with the fresh water through a series of heat exchangers. The fuel gas leaves the chimney at approximately 200 °C. Fig. 2 shows the chemical recovery boiler used in this study.

The collected data set is composed of nineteen process variables as shown in Table 1. There are fifteen input variables, one intermediate variable (column 1; variable 16) and three output variables (column 1; variables 17-19). The collection of data includes four months of operation with an hourly sampling interval, which leads

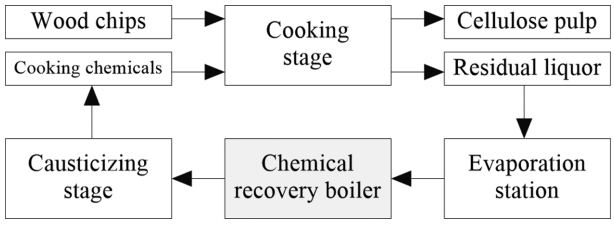

Fig. 1. Main stages of Kraft pulp mills.

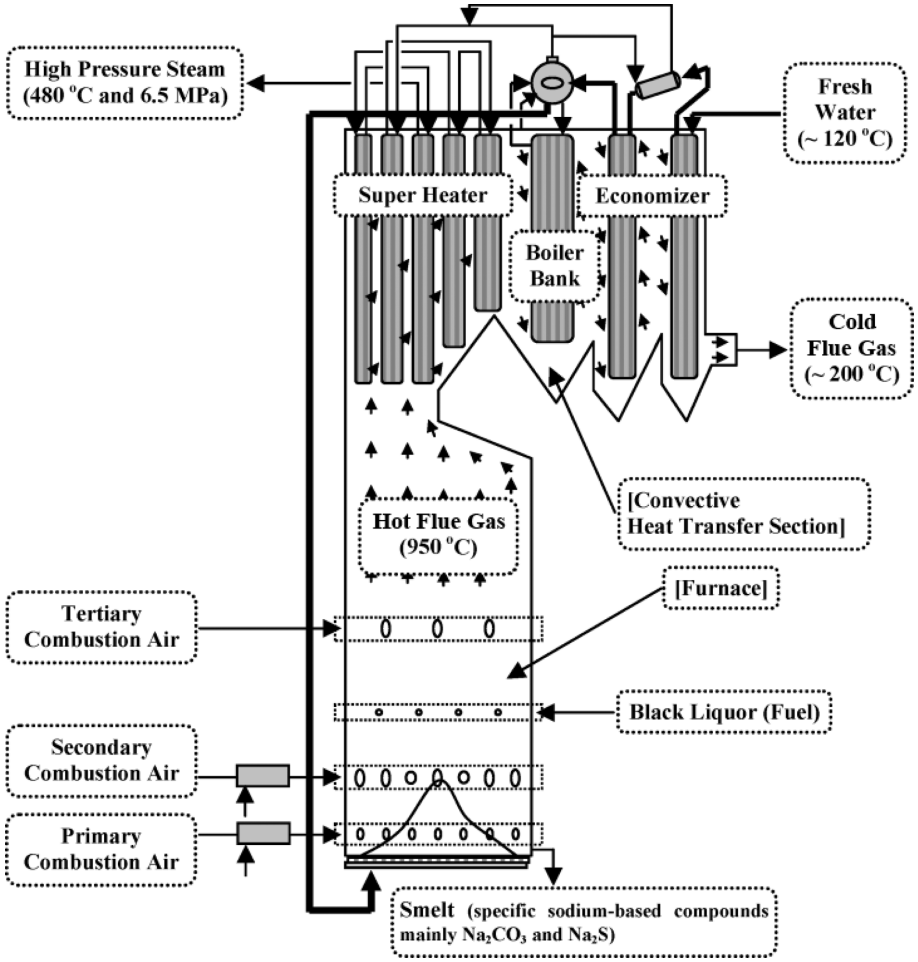

Fig. 2. Scheme of the chemical recovery boiler of the case study.

Table 1. Data set of the case study

Number	Variable	Unit	Mean	Standard deviation
1	Fuel flow rate (residual liquor)	ton/h	109.4	9.1
2	Fuel temperate	°C	126.4	1.0
3	Fuel pressure (walls 2 and 4)	mmH_2O	0.855	0.077
4	Fuel pressure (walls 1 and 3)	mmH_2O	0.902	0.070
5	Dry solids content (measure 1)	%	68.2	1.7
6	Dry solids content (measure 2)	%	68.2	1.6
7	Primary air flow rate	ton/h	153.9	7.0
8	Primary air temperature	°C	150.0	1.9
9	Primary air pressure	mmH_2O	37.4	7.0
10	Secondary air flow rate	ton/h	187.5	21.0
11	Secondary air temperature	°C	167.0	3.9
12	Secondary air pressure	mmH_2O	202.8	18.5
13	Tertiary air flow rate	ton/h	48.4	3.0
14	Tertiary air temperature	°C	29.9	4.6
15	Tertiary air pressure	mmH_2O	200.2	17.1
16	Boiler drum pressure	kg/cm^2	73.5	1.9
17	H_2S emissions	ppm	0.266	0.980
18	SO_2 emissions	ppm	119.908	107.587
19	Steam flow rate	ton/h	297.9	26.5

to 2,928 measurements for each variable. The operating condition of the boiler is considered normal throughout the period. For benchmark processes, it is easy to get normal data. However, when working with real processes, this evaluation should be supported by experienced operators and engineers. This point was previously verified in this work through a visual inspection of the whole data set and by an evaluation of the mill process team, from which the data set was considered characteristic of normal operations.

STOCHASTIC OPTIMIZATION OF THE BOILER PROCESS

1. The Modeling of a Boiler Process

To address the qualitative models associated with steam generation of the boiler, a general discrete-time dynamical system including manipulated and input variables can be described by Eq. (1),

$$\begin{aligned} &BP(t+1)=g(x(t), u(t), BP(t)) \\ &Y_{Steam}(t+1)=h_1(x(t), u(t), BP(t+1)) \\ &Y_{SO_2}(t+1)=h_2(x(t), u(t), BP(t+1)) \\ &Y_{H_2S}(t+1)=h_3(x(t), u(t), BP(t+1)) \end{aligned} \quad (1)$$

where x(t) represents the input vector, u(t) is the vector including manipulated variables, BP(t) is the intermediate variable (that is, the boiler drum pressure), and y(t) is the output vector. There are three output variables (Y): the steam flow rate and the SO_2 and H_2S levels of emissions.

The prior process knowledge based on the first-principle equations is unknown for this system regarding mainly its operational dynamics as a whole, and more specifically the emissions of gases. Thus, the functions g and h are built through empirical models.

SVR is one of the most effective regression techniques for handling nonlinear information [14]. First, the inputs are projected into a high-dimensional feature space by means of a kernel function. This information is then correlated with the system output. SVR has the advantage of high predictive capability as minimizing model complexities [25]. Eq (2) presents its overall regression function, where w is the weight vector, x is the input data, ϕ is the kernel function, and b is the bias.

$$f(x, w)=w^T\Phi(x)+b \quad (2)$$

The parameter estimation is given by minimizing Eq. (3), in which $||w||^2$ is the model complexity, ξ is the error tolerance, and C is a scalar value to control the trade-off between both of them [14]. In this study, the ε-insensitive loss function is adopted to ensure that a error greater than ε is penalized as shown in Eq. (4).

$$\min_{w, b, \xi} \frac{1}{2}\|w\|^2 + C\sum_{i=1}^{m}\xi_i \quad (3)$$

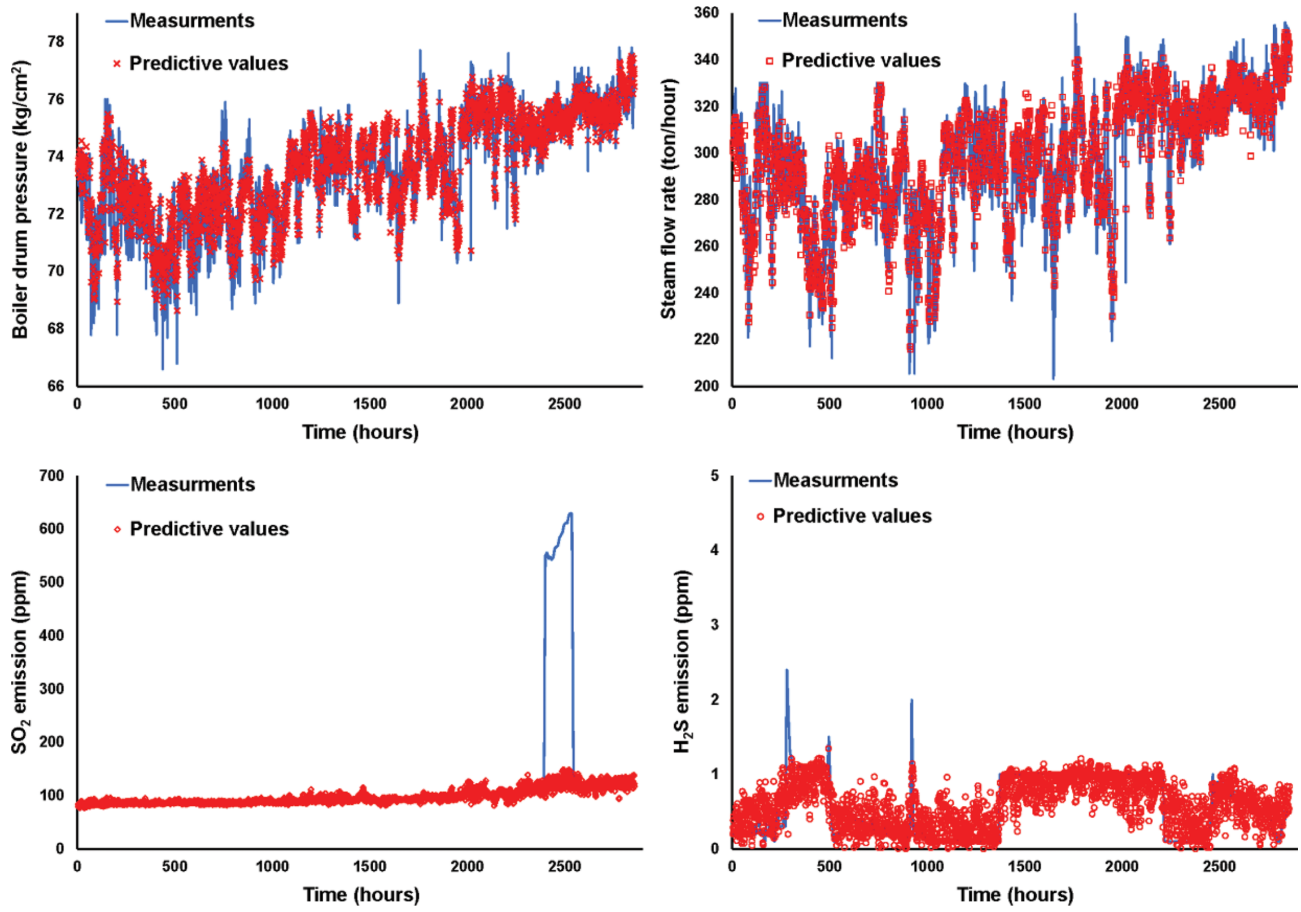

Fig. 3. Comparison between mill measurements (blue line) and model estimates (red lines) for the boiler pressure drum, steam flow rate, and SO_2 and H_2S emissions.

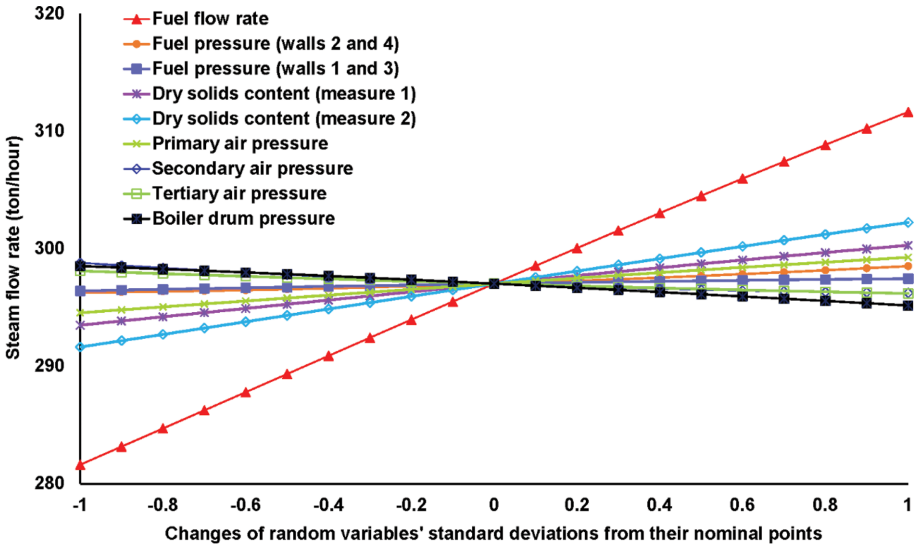

Fig. 4. Sensitivity analysis study over the steam flow rate variable.

Table 2. Sensitivity analysis study: Rank of the effects of the input variables over the steam flow rate variable

Random variable	Nominal value±σ	Absolute average slope [ton/(hour·σ)]	Rank
Fuel flow rate	109.40±9.13	1.50	1
Fuel pressure (Wall 2&4)	0.90±0.08	0.11	7
Fuel pressure (Wall 1&3)	0.90±0.07	0.05	9
Dry solids content (Measure 1)	68.20±1.66	0.34	3
Dry solids content (Measure 2)	68.20±1.62	0.53	2
Primary air pressure	37.40±7.04	0.23	4
Secondary air pressure	202.80±18.54	0.13	6
Tertiary air pressure	200.20±17.11	0.10	8
Boiler drum pressure	73.50±1.91	0.17	5

subject to

$$\xi_i = \begin{cases} |y_i - f(x_i, w)| - \varepsilon, & \text{if } |y_i - f(x_i, w)| \geq \varepsilon \\ 0, & \text{otherwise} \end{cases} \quad (4)$$

This work used the radial basis function (RBF) as the kernel function, where σ is a free parameter in Eq. (5). In RBF-based SVR, C, ε and σ should be specified by the user. In this study, these values are optimized by using the PSO technique. Detailed concepts and equations regarding SVR may be found in Vapnik [14].

$$K(x_i, x_j) = (\phi(x_i) \cdot \phi(x_j)) = \exp\left(-\frac{|x - x|}{2\sigma^2}\right) \quad (5)$$

Fig. 3 shows the results of four regression models, whose parameters are optimized by PSO. These models can be applied to find the stochastic optimal design variables. Regarding the SO_2 prediction, it can observe values above 500 ppm. They are associated with a sensor fault. Except these, most estimates are satisfactorily close to the mill measurements.

2. Sensitivity Analysis of Random Variables

To identify how much each random variable affects the objective function, a sensitive analysis was carried out. Due to their different scales, all random variables were normalized. The sample points of normalized random variables were selected with equally spaced intervals from their nominal values. The changes in the objective function were evaluated according to ±100% changes in the standard deviations of the random variables ($-\sigma$, -0.8σ, -0.6σ, -0.4σ, -0.2σ, 0, 0.2σ, 0.4σ, 0.6σ, 0.8σ and σ). A specified random variable was investigated at a time while keeping the remaining variables fixed in their respective nominal values.

Fig. 4 shows the results of this sensitivity analysis study. The impact of each random variable is depicted in Table 2 in a quantitative way. It can be observed that four variables present larger impacts on the objective function (last column; Ranks 1-4). The next section describes the strategy for building a stochastic optimization problem.

SAMPLE AVERAGE APPROXIMATION

The aim of stochastic optimization is to find the optimal values for decision variables. Given the consideration of random variables, it yields to the best average performance of the objective function. A stochastic optimization may provide more realistic solutions than a deterministic optimization, depending on the strength of the impact of the random components [16,22]. In stochastic optimization approaches, highly complex nonlinear models are generally decom-

posed into a few linear models, making use of their convexity and differentiability. In case of variable load, more than one linearization procedure might be needed. Unlike, the boiler of the case study operates close to its maximum capacity rate (MCR) throughout the period.

In this study, the SVR-based objective functions are black box models; hence, mathematical equations are not revealed. This fact makes the adoption of the general stochastic optimization approach impossible.

To solve this problem, several researchers have studied approximation methods. They can be divided into two major approaches [26]. The first decomposes the feasible sample spaces, and the convergence around the optimal solutions occurs as iterations increase. However, for decomposition, the differentiability of constraints should be available since it is necessary to obtain their convexity and monotonicity. Since the models in this study are not explicit, the differentiability of the constraints is not available.

The second approach, Monte-Carlo sampling-based sample average approximation (SAA), is used. This technique generates scenarios through the combination of the sampling points of a random variable set, and the optimal solution is then investigated [16,22]. This means that this solution provides the best performance of the objective function, on average, in the scenarios. This can be easily used without the need for differentiability, even if the model is very complex and highly nonlinear. Since it may suffer from the curse of dimensionality, a huge amount of computational processing is required, which means that the number of scenarios may grow exponentially. The key issue is how to reduce the number of scenarios to avoid the curse of dimensionality if the random variable subset is relatively large. This can be formulated as in Eq. (6);

$$\hat{f}_N(Q) = \frac{1}{N}\sum_{k=1}^{N} f(Q, \varepsilon_k) \tag{6}$$

where f is the objective function, Q is the set of design variables in all possible scenarios, ε_k indicates each scenario, and N is the total number of scenarios. The goal is to find the best Q solution.

In this study, it is not feasible to employ gradient-based optimization solvers to find the optimal Q, since the objective function is an empirical and not explicit model regarding the boiler process. For this purpose, PSO, a gradient-free optimization solver, was employed. This solver may handle empirical black box or highly nonlinear models.

According to the sensitivity analysis results, the variable subset with the most significant effects on the objective function was identified. To control the number of scenarios, the number of sampling points from negligible variables was reduced.

PARTICLE SWARM OPTIMIZATION (PSO)

Due to the infeasibility of employing a gradient-based optimization solver, the stochastic optimization problem in this study was solved with the original PSO technique. This is a representative population-based heuristic optimization technique, and so without the need for explicit derivatives [27]. First, a population of particles, called a swarm, is randomly generated on an *N*-dimensional search space. The position and velocity of the particles are then updated according to the historical best solutions [28]. At the end, they converge to the best optimal solution. The original PSO can be easily applied to black-box models where governing equations are not revealed. Additionally, it is a relatively small parameter set in comparison with other population-based heuristic optimization techniques such as the genetic algorithm. More detailed concepts and steps can be found in the work of Schwaab et al. [29].

THE OBJECTIVE FUNCTION

In this study, the stochastic optimal solution is defined as concerning the minimization of the expected value of an objective function in scenarios generated by random variables. The objective function (F) is given by Eq. (7);

$$\begin{aligned}
&\max_u \mathbb{E}[F(x(t)), u, \varepsilon] \\
&\text{subject to} \\
&C_i \begin{cases} Y_i(x(t), u, BP(t+1)) - R_i, & \text{if } Y_i - R_i \geq 0 \\ 0, & \text{otherwise} \end{cases} \\
&F(x(t), u, BP(t+1)) = Y1 - W_{SO_2} \cdot C_{H_2S} - W_{H_2S} \cdot C_{H_2S} \\
&\mathbb{E}[F(x(t), u, \varepsilon)] = \frac{1}{N}\Sigma_{t=1}^{T}\Sigma_{k=1}^{N} F(x(t), u, \varepsilon_k)
\end{aligned} \tag{7}$$

R_i is the limit value of emissions for chemical compound *i*, and C_i ensures a penalty when a predictive emission for *i* is larger than this threshold. W is the weighting value to penalize the violation of emission regulations; ε_k is a scenario generated by the combination of sampling points, and N is the number of scenarios; u(t) is the set of decision variables; and $\mathbb{E}[F(u, \varepsilon)]$ is the expected value of the objective function given all scenarios. In this study, two hundred sets of decision variables were randomly generated, and the expected values of the objective function were then evaluated. They were updated via PSO at each iteration, and the stochastic optimal solution was finally reached.

RESULTS AND DISCUSSION

The stochastic optimization problem for a boiler process was constructed to maximize the expected value of an objective function under all scenarios generated by the combination of random samples. From the previous sensitivity analysis results (Section 3.2), the variables which had significant effects on the objective function were identified. To control the number of scenarios, the number of sampling points on negligible variables was reduced, and in the opposite direction they were increased for the significant variables. Five sampling points were randomly generated for the fuel flow rate, dry solid contents (measures 1 and 2) and primary air pressure, and three samples were randomly generated for the other variables. It was assumed that all random variables followed the Gaussian distribution. The total number of scenarios was $5^4 \times 3^5 = 151{,}875$. For 100 sets of decision variables randomly generated and updated through PSO, 1.51875×10^7 calculations should be accomplished at each iteration as well as 100 expected values of the objective function. This study worked with 200 sets yielding 3.03750×10^7 calculations every iteration. An unappreciated fact of a SAA-based stochastic optimization is that a global stochastic solution

cannot be found. To overcome this disadvantage, a series of simulations were performed. The CPU time for one iteration, with an AMD Athlon 2.9 GHz, was equal to 1,013,234 seconds. This combinatorial optimization problem is shown in Eq. (8) where ε is the set of scenarios generated by sampling points on random variables.

$$\max_u \Sigma_{t=1}^{2,928} \Sigma_{k=1}^{151,875} E[F(x(t), u, \varepsilon_k)] \quad (8)$$

To verify the efficacy of the stochastic optimization, a deterministic solution was also investigated. In this case, all random variables are set to their respective nominal values. The values of the objective function for both approaches were then evaluated given the data set of the boiler process. Table 3 summarizes both the stochastic and the deterministic optimal solutions, and Table 4 shows the optimal values for both objective functions given by Eqs. (9) and (10), respectively. It can be observed that the objective function value of the stochastic solution was higher. Since the deterministic solution ignored real changes in the random variables, the stochastic optimal solution provided better results in real scenarios. There were no violations concerning the levels of SO_2 and H_2S emissions in both approaches.

$$u^{d*} = \arg_u \max \mathbb{E}[F(x(t), u, \mathbb{E}(\varepsilon))] \quad \text{Deterministic solution} \quad (9)$$

$$u^{s*} = \arg_u \max \mathbb{E}[F(x(t), u, \varepsilon)] \quad \text{Stochastic solution} \quad (10)$$

In this study, the value of the stochastic solution (VSS), given by Eq. (11), is introduced. The positive value, equal to 5.880, means that the stochastic solution's average performance is better than the deterministic one. This difference can be seen in Fig. 5 under real scenarios. To evaluate its efficacy more accurately, a paired statistical hypothesis *t*-test considering the difference between both solutions, was employed [30]. Eq. (12) shows the formulation, in which

Table 3. Result of the stochastic and deterministic optimization approaches: Final design variable values

Design variable	Stochastic solution	Deterministic solution
Fuel temperature	125.47 °C	125.46 °C
Primary air flow rate	147.49 ton/h	146.94 ton/h
Primary air temperature	148.16 °C	148.08 °C
Secondary air flow rate	205.36 ton/h	208.47 ton/h
Secondary air temperature	163.10 °C	163.06 °C
Tertiary air flow rate	51.08 ton/h	51.47 ton/h
Tertiary air temperature	34.34 °C	34.47 °C

Table 4. Results of the stochastic and deterministic approaches: Final objective function values

Result	Stochastic solution	Deterministic solution
An average value of an objective function	307.252 ton/h	301.373 ton/h
Standard deviation	20.680	22.808

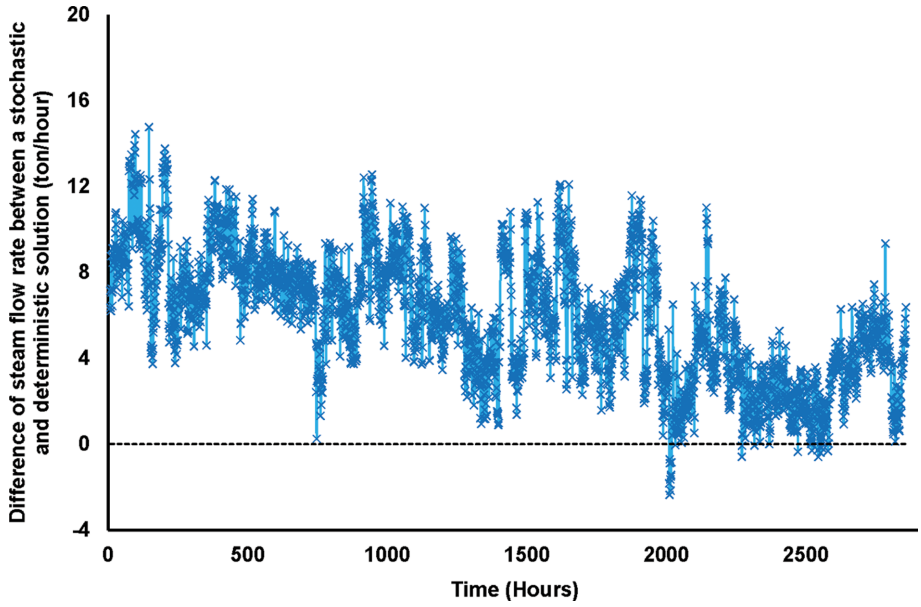

Fig. 5. Differences between the stochastic and the deterministic optimal solutions for the steam flow rate (Positive values indicate the best performance of the stochastic optimal solution.).

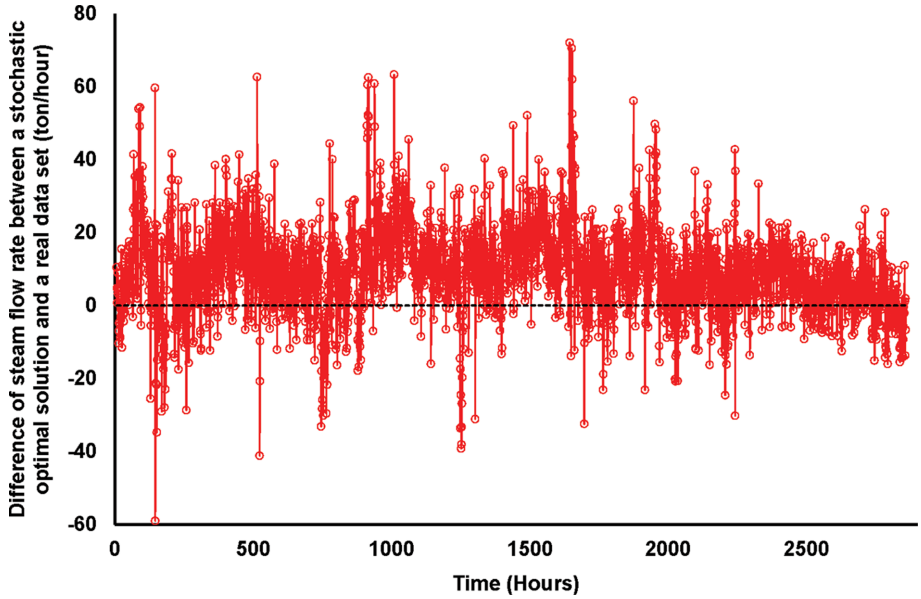

Fig. 6. Comparison between the stochastic optimal solution with the real data set of the steam flow rate (Positive values indicate the performance of the stochastic optimal solution is better than that of a real data set).

μ_d and s_d are, respectively, the mean and the standard deviation of the paired variable, and N is the number of scenarios. A significance level (α) of 1% was adopted, i.e., the critical limit, $t_{0.01/2}$, is equal to 2.58. The confidence interval for μ_d was given by 5.880±0.103, with a lower limit of 5.777. These results show that the stochastic optimal solution provided higher steam production by at least 1.914% in relation to the deterministic solution. Moreover, the steam flow rate provided by the stochastic solution was 3.17% greater than the mean real value. Fig. 6 shows that its performance surpassed the mill values with respect to boiler steam generation. Therefore, the stochastic optimal solution considering random variables presented the best performance without emission violation.

$$\begin{aligned} VSS &= \mathbb{E}_{\varepsilon}[F(x(t), u^{s^*}, \varepsilon)] - \mathbb{E}_{\varepsilon}[F(x(t), u^{d^*}, \varepsilon)] \\ &= 307.25 - 301.37 = 5.880 \end{aligned} \tag{11}$$

$$-t_{\frac{0.01}{2}} \le \frac{VSS - \mu_d}{\frac{s_d}{\sqrt{N}}} \le t_{\frac{0.01}{2}} \tag{12}$$

CONCLUSIONS

Four discrete models were set, since this number is enough to satisfactorily predict the process target variables according to the available process knowledge. Conventional simulators are not able to mimic the reboiler process; therefore SVR was employed to make four regression models and used to find the optimal operational condition. A set of SVR models were identified with the aim of representing the boiler process and the environmental restrictions. To maximize steam generation and avoid violating regulations associated with pollutant emissions, the stochastic optimization problem was generated due to the existence of random variables. Because the mathematical models of SVR were not revealed, SAA based on Monte-Carlo sampling was implemented. Moreover, since there are many unknown physicochemical phenomena and the derivatives of SVR models were not available, a gradient-free based PSO technique, which is easily combined with empirical models, was used to investigate a stochastic optimal solution. To verify the efficacy of a stochastic optimal solution, VSS was evaluated based on the deterministic solution. The results show that steam generation under a stochastic optimal solution was more improved than that under real data conditions. Furthermore, VSS values indicate that the stochastic optimal solution provided more steam generation than the deterministic solution ignoring random variables. Therefore, a stochastic solution may provide more accurate results in comparison to the deterministic approach by taking into account the stochastic nature of the process variables. Because the proposed method is based on SVR, which is a black box model, and PSO, which is a gradient-free technique, it can be easily applied to various problems including various random variables.

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