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A reliability model for process systems under changing operating conditions

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Abstract−Reliability analysis of process systems, which is often based on a model of Weibull distribution, is semi-quantitative at best because it uses constant parameters, requiring assumption of steady state operating conditions. A reliability model based on a variable scale parameter Weibull distribution is proposed in this work, in which a power law, the Arrhenius factor, and instantaneous amplitudes and frequencies of the operating condition variables are introduced. Numerical experiment indicates that when an operating condition variable fluctuates, the assumption of an average steady state operating condition can cause a serious error in reliability analysis. Therefore, the proposed method is expected to contribute to more quantitative risk assessment, and thus more rigorous safety analysis of process systems under changing operating conditions.

Keywords: Reliability Model, Process Safety, Operating Condition, Weibull Distribution, Instantaneous Amplitude, Instantaneous Frequency

INTRODUCTION

Process safety requires reliability analysis of process facilities and devices, for which the method proposed by API (American Petroleum Institute) [1] may be considered a de facto standard. The API method, however, includes oversimplifications, which have encountered criticism [2], and a part of the method has been modified [3]. The main issue is that the probability of failure cannot be predicted by statistical theory only. The history of operating conditions should be considered. The API method, as will be shown later, does this only semi-quantitatively, and a fully quantitative method is to be proposed in this work.

Reliability analysis frequently uses a reliability (survival) function, which represents the probability that the object will be alive at a moment of time in the future. The most widely used model for this function is the Weibull distribution [4] defined as follows:

$$R(t)=\exp\left[-\left(\frac{t}{\eta}\right)^{\beta}\right] \tag{1}$$

where η is the scale parameter, and β is the shape parameter. Generally, for a specific device, η depends on the operating condition, and β can be assumed to be constant. For example, Table 1 shows the default values of these parameters for conventional pressure relief valves for the probability of failure on demand and the probability of leakage provided by API [1]. Table 2 shows how the API method adjusts the scale parameter according to different operating conditions. Now the problem is how to modify the scale parameter when the operating conditions change with time.

Generally, the reliability function continuously and smoothly decreases from 1 to 0, even if the operating conditions change with time. However, device life can be extended by maintenance, which means that the probability of failure (death) can be decreased by maintenance. In this case, the reliability function should be reset or updated. Inspection can also affect the reliability function. A Bayesian method to update the scale parameter based on the confidence factor of the inspection is also provided by API [1].

A device life depends on the operating conditions. The acceleration by thermal stress is typically modeled by the Arrhenius equation as follows [5]:

$$AF=\exp\left[-\frac{E_a}{k_B}\left(\frac{1}{T}-\frac{1}{T_0}\right)\right] \tag{2}$$

Table 1. Default Weibull parameters for conventional pressure relief valves [1]

Fluid severity	Failure on demand		Leakage	
	η (years)	β	η (years)	β
Mild	50.5	1.8	17.5	1.6
Moderate	23.9	1.8	15.5	1.6
Severe	17.6	1.8	13.1	1.6

Table 2. Adjustment of Weibull η parameter for operating conditions [1]

Environment modifier	Adjustment factor	
	Failure on demand	Leakage
T<200 °F	1.0	1.0
200<T<500 °F	1.0	0.8
T>500 °F	1.0	0.6
Operating ratio >90%	1.0	0.5
Installed piping vibration	1.0	0.8
Pulsating or cyclical service	1.0	0.8
History of actuation >5 times/yr	0.5	0.5
History of chatter	0.5	0.5

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where AF is the acceleration factor, T_0 is the reference temperature, E_a is the activation energy, and k_B is the Boltzmann constant.

The pressure stress on process systems such as pressure safety devices can be considered analogous to the voltage stress on the insulations in electronic devices. The life acceleration by voltage stress is often modeled by a power law as follows [5]:

$$AF = \left(\frac{U}{U_0}\right)^{\alpha} \tag{3}$$

where U and U_0 represent the voltage at test and use conditions, respectively, and $\alpha>0$.

The device life is also affected by fluctuations in the operating conditions. For example, the solder fatigue in electronic devices can be modeled by the following modified Coffin-Manson equation, i.e. Norris-Landzberg model [6]:

$$AF = \frac{N_f}{N_t} = \left(\frac{f_t}{f_f}\right)^m \left(\frac{\Delta T_t}{\Delta T_f}\right)^n \exp\left[-\frac{E_a}{k_B}\left(\frac{1}{T_{t,\, max}} - \frac{1}{T_{f,\, max}}\right)\right] \tag{4}$$

where AF represents the acceleration factor of the device life on a cycle basis for the test condition, T_t, over the field condition T_f, N is the number of cycles survived, f is the frequency, and ΔT is the amplitude of the oscillation of the temperature. For SnPb solder, $m=-1/3$, $n=1.9$, $E_a/k_B=1414$ K. Note that if we redefine the acceleration factor in life time instead of life cycles, $AF=(N_f/f_f)/(N_t/f_t)$, the exponent m is replaced by $m_1=m+1$, i.e. $m_1=2/3$ for SnPb solder.

PROPOSED METHOD

For changing operating conditions, the Weibull distribution (1) cannot be used because the scale parameter, which depends on the operating condition, is constant. As a solution, an extended Weibull distribution, in which the scale parameter is a function of time, was proposed in the author's previous work, which can be described as follows [7]:

$$R(t)=\exp[-z(t)^{\beta}] \tag{5}$$

where z(t) is the dimensionless time that represents aging, defined as follows [7]:

$$z(t) = \int_0^t \frac{dt}{\eta(t)} \tag{6}$$

where $\eta(t)$ represents the scale parameter for the operating condition at time t. In the previous work, the scale parameter was simply a function of operating condition variables: it depended on the values of the variables only. However, aging occurs by fluctuations also. Therefore, an extended scale parameter model is proposed in this work, which depends on the operating condition variables and their fluctuations as follows:

$$\eta(t)=\eta(\mathbf{v}(t), \mathbf{A}(t), \mathbf{F}(t)) \tag{7}$$

where $\mathbf{v}(t)$ is a vector of operating condition variables, and $\mathbf{A}(t)$ and $\mathbf{F}(t)$ are matrices of the instantaneous amplitudes (IA) and the instantaneous frequencies (IF), respectively, involved in $\mathbf{v}(t)$. If the operating condition is constant, the original Weibull distribution is retained.

The instantaneous amplitudes and frequencies can typically be obtained by Hilbert-Huang transform (HHT). First, a given function $v_i(t)$ is decomposed into a sum of intrinsic mode functions (IMF), which are functions that oscillate symmetrically around zero, plus a residue, using the empirical mode decomposition (EMD) method proposed by Huang et al. [8] as follows:

$$v_i(t) = \sum_{j=1}^{k_i} x_{ij}(t) + r_i(t) \tag{8}$$

where $x_{ij}(t)$ is the j-th IMF, and $r_i(t)$ is the residue which represents the trend of $v_i(t)$ [9]. For example, if $v_i(t)$ linearly changes with oscillations, $r_i(t)$ is a linear function, and $x_{ij}(t)$'s are trigonometric functions.

For an IMF $x_{ij}(t)$, the IA and IF can be obtained by Hilbert transform defined as follows:

$$y_{ij}(t) = \frac{1}{\pi} P\int_{-\infty}^{\infty} \frac{x_{ij}(\tau)}{t-\tau} d\tau = -\frac{1}{\pi}\lim_{\varepsilon\to 0^+}\int_{\varepsilon}^{\infty} \frac{x_{ij}(t+\tau)-x_{ij}(t-\tau)}{\tau} d\tau \tag{9}$$

where P represents the Cauchy principal value. The instantaneous amplitude, phase, and frequency can be obtained respectively as follows:

$$a_{ij}(t) = \sqrt{x_{ij}(t)^2 + y_{ij}(t)^2} \tag{10}$$

$$\phi_{ij}(t) = \tan^{-1}\frac{y_{ij}(t)}{x_{ij}(t)} \tag{11}$$

$$f_{ij}(t) = \frac{1}{2\pi}\frac{d\phi_{ij}}{dt} \tag{12}$$

There are other methods to evaluate instantaneous amplitude and frequency, which are actively studied in many areas that require signal processing.

Let us assume that the service life acceleration of process systems can be modeled by a power law and/or the Arrhenius equation in terms of the operating condition variables, and that there is no interaction between the operating condition variables. Then, the following model is proposed:

$$\frac{1}{\eta(t)} = \frac{1}{\eta_0}\prod_i \left[\frac{v_i(t)}{v_{i0}}\right]^{\alpha_i} \exp\left\{-\frac{E_{a,i}}{k_B}\left[\frac{1}{v_i(t)} - \frac{1}{v_{i0}}\right]\right\} AF_i\left(\sum_{j=1}^{k_i}\left[\frac{a_{ij}(t)}{a_i^o}\right]^{m_i}\left[\frac{f_{ij}(t)}{f_i^o}\right]^{n_i}\right) \tag{13}$$

where η_0 represents the scale parameter at the reference operating condition $\mathbf{v}_0$, and AF_i is a continuously increasing function such that $AF_i(0)=1$, $AF_i(1)=AF_i^o$, and $AF_i(x)\to AF_i^o x$ as $x\to\infty$, where AF_i^o (>1) is the reference acceleration factor for oscillation with amplitude a_i^o and frequency f_i^o. Also, $a_{ij}(t)$ and $f_{ij}(t)$ represent the IA and IF of the *j*-th IMF component of $v_i(t)$ respectively.

There are various ways to model function AF_i, and the following piecewise cubic spline is proposed in this work:

$$AF_i(x) = \begin{cases} (2-AF_i^o)x^3 + (2AF_i^o-3)x^2 + 1, & \text{if } 0\le x<1 \\ AF_i^o x, & \text{if } x\ge 1 \end{cases} \tag{14}$$

This function is smooth at x=1, and satisfies all the requirements mentioned above if and only if $AF_i^o \ge 3/2$.

For process systems, typical operating condition variables are

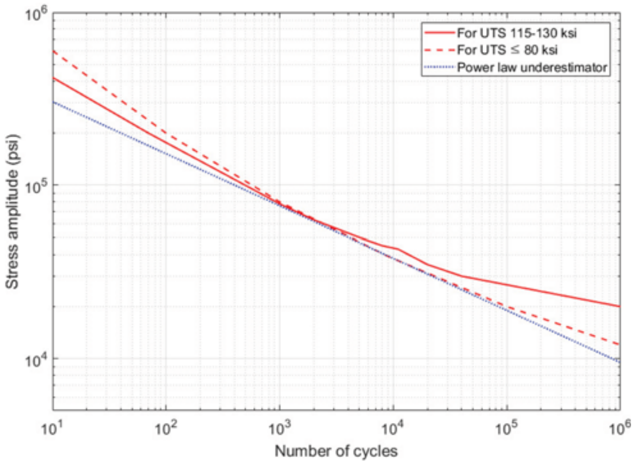

Fig. 1. Stress vs. number of cycles curves for pressure vessels [10].

temperature and pressure. If $v_i(t)$ in the above equation is temperature, it is expected that $\alpha_i\approx 0$ $(0\le\alpha_i<1)$, and $E_{a,i}$ can be obtained from life vs. temperature data. For temperature fluctuations, m_i and n_i can be estimated if the Norris-Landzberg model can be applied. If $v_i(t)$ is pressure, it is expected that a power law applies, and the Arrhenius factor is unnecessary: $E_{a,i}=0$. Furthermore, if it is assumed that life acceleration is mostly by fluctuation rather than the pressure itself, it is also expected that $\alpha_i\approx 0$. This also implies $n_i\approx 1$. The parameter m_i can be obtained from a stress vs. number of cycles curve (S-N curve). Fig. 1 is an example for pressure vessels obtained from ASME (American Society of Mechanical Engineers) data [10]. Each curve in the figure represents the maximum pressure amplitude vs. the number of cycles that the vessel can survive. The straight line in the figure is a tight power law underestimator: a conservative model for estimating the life of the vessel. The slope of this line indicates that if the amplitude is doubled ($2\times10^4\rightarrow4\times10^4$ psi) the number of cycles decreases to one-tenth ($9\times10^4\rightarrow9\times10^3$). Life time is proportional to the number of cycles if the period is constant. Therefore, the acceleration factor in this case is $2^{m_i}=10$, hence $m_i=1/\log 2\approx 3.3$.

NUMERICAL EXPERIMENT

1. System Description

The proposed model is applied to a conventional pressure relief valve for the probability of leakage. Let us assume that the fluid severity is moderate, the operating temperature is 450±100 K (≈350±180 °F), the operating pressure is 5±1 MPa, and the relief set pressure is 6 MPa. The default Weibull parameters are obtained from Table 1. Among these, the shape parameter is to be adjusted according the operating conditions as shown in Table 2.

2. Modeling

The effect of the operating temperature on the scale parameter is listed in Table 3. Based on this semi-quantitative information, it is assumed that η=12.4 at T=350 °F, $\eta\approx$15.5 at T=170 °F, and $\eta\approx$9.3 at T=530 °F. Fitting these data to the Arrhenius model results in $E_a/k_B\approx$400 K.

The effects of fluctuations in the operating pressure on the scale

Table 3. Adjustment of Weibull η parameter for operating temperatures [1]

Temperature (°F)	Adjustment factor	η (years) for moderate fluid severity
T<200	1.0	15.5
200<T<500	0.8	12.4
T>500	0.6	9.30

Table 4. Adjustment of Weibull η parameter for fluctuations in pressure [1]

Fluctuation type	Adjustment factor	η (years) at T=350 °F
Installed piping vibration	0.8	9.92
Pulsating or cyclical service	0.8	9.92
History of actuation >5 times/yr	0.5	6.20
History of chatter	0.5	6.20

parameter are listed in Table 4. The information is just qualitative except the third one, from which it is assumed that η decreases to half if the relief set pressure is reached six times per year. As the power law of the pressure and the influence of the temperature fluctuation are unknown at this point, it is also assumed that $\alpha_P\approx 0$, $AF_T^o\approx 1$, and $m_T\approx n_T\approx 0$. Therefore, the following model is constructed:

$$\frac{1}{\eta(t)}=\frac{1}{\eta_0}\exp\left\{-\frac{E_a}{k_B}\left[\frac{1}{T(t)}-\frac{1}{T_0}\right]\right\}AF_P\left(\sum_{j=1}^{k_P}\left[\frac{a_{P_j}(t)}{a_P^o}\right]^{m_P}\left[\frac{f_{P_j}(t)}{f_P^o}\right]^{n_P}\right) \tag{15}$$

where η_0=12.4 yrs, T_0=450 K, E_a/k_B=400 K, AF_P^o=2, a_P^o=1 MPa, f_P^o=6 yr^{-1}, m_P=3.3, and n_P=1. The values of m_P and n_P have been borrowed from the pressure vessel case discussed above, as S-N curves for pressure relief valves are unavailable at this point.

3. Case Study

The following four cases of operating conditions are tested in this work.

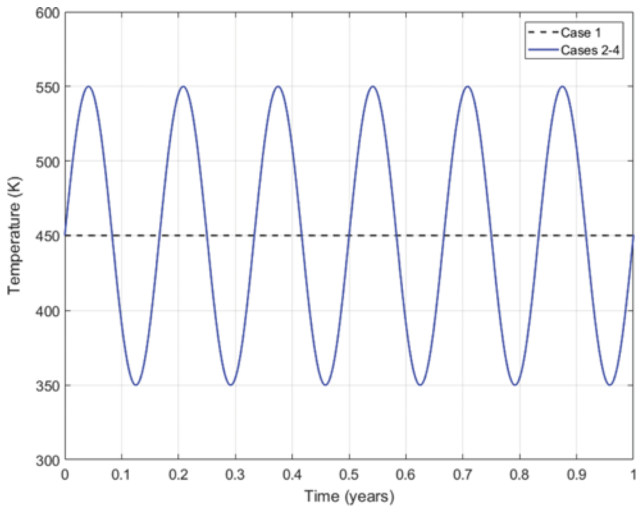

Fig. 2. Operating temperatures.

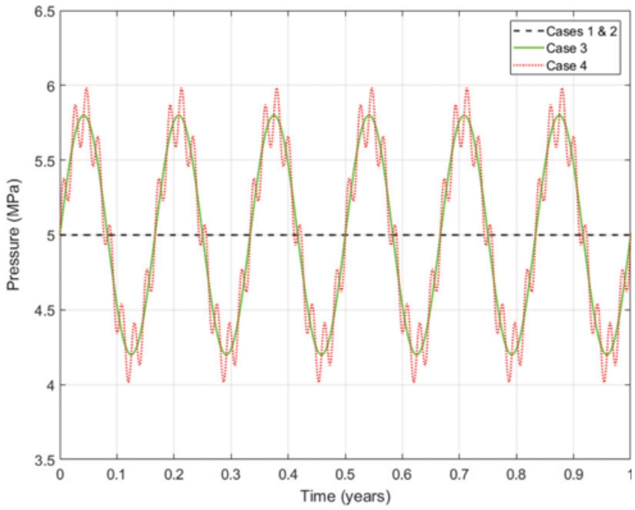

Fig. 3. Operating pressures.

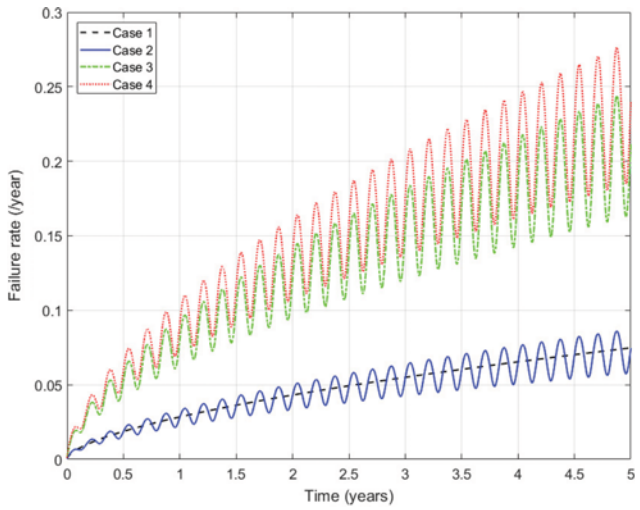

Fig. 5. Failure rate functions.

Case 1. T(t)=450, P(t)=5
Case 2. T(t)=450+100 sin12πt, P(t)=5
Case 3. T(t)=450+100 sin12πt, P(t)=5+0.8 sin12πt
Case 4. T(t)=450+100 sin12πt, P(t)=5+0.8 sin12πt+0.2 sin96πt

Figs. 2 and 3 show the operating temperatures and pressures respectively. For pressures, HHT is to give results close to the following data:

Case 3. k_P=1, a_{P1}(t)=0.8 MPa, f_{P1}(t)=6 yr^{-1}
Case 4. k_P=2, a_{P1}(t)=0.8 MPa, f_{P1}(t)=6 yr^{-1},
a_{P2}(t)=0.2 MPa, f_{P1}(t)=48 yr^{-1}

Fig. 4 shows the reliability (survival) function. Note that the curve of case 2 is slightly higher than that of case 1. This means that temperature fluctuation results in a longer life, which is unacceptable, indicating that the Arrhenius factor cannot work alone as acceleration factor when temperature fluctuates. The figure indicates that pressure fluctuation significantly increases the probability of failure F(t)=1−R(t). For example, the probability of leakage in 2 years is about 5% in cases 1 and 2, and about 15% in cases 3 and 4.

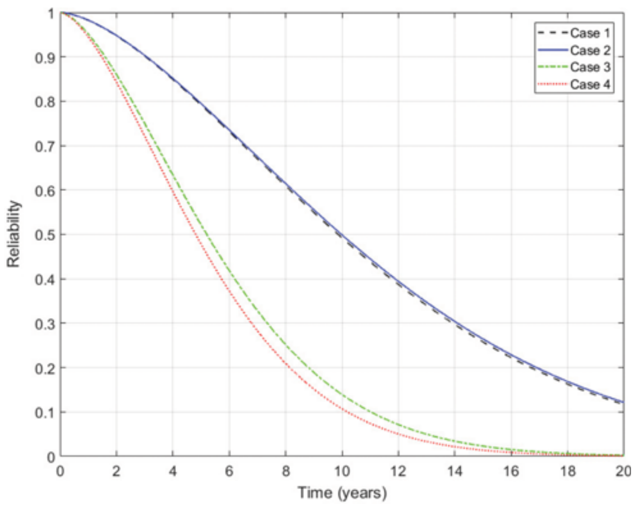

Fig. 4. Reliability functions.

Fig. 5 shows the failure rate function, which is defined as follows:

$$\lambda(t) = -\frac{R'(t)}{R(t)} \tag{16}$$

For safety, the estimated failure rate should be maintained at or below an acceptable level. Let us assume that the acceptable maximum frequency of leakage of a pressure safety valve in a plant is 0.05/yr, so once in 20 years. The figure indicates that the valve can be used for 2.5 years if operated at a steady state as in case 1, but only for 4 months if the operating pressure fluctuates as in case 3 or 4.

CONCLUSIONS

Process safety is often based on layer of protection analysis (LOPA), which is semi-quantitative [11]. Quantitative risk assessment (QRA) is required as the core technology of process safety, not only in process operation, but also in plant design [12]. QRA requires reliability analysis, but the conventional API method [1] is not fully quantitative.

A reliability model has been proposed based on an extended Weibull distribution in which the scale parameter is a function of time that depends on the operating condition variables and their instantaneous amplitudes and frequencies. Numerical experiment indicates that pressure oscillations can significantly decrease the reliability, and increase the failure rate.

However, data are insufficient for practical applications. For process safety, more process data should be collected and organized in such a way that the influence of fluctuations in the operating conditions can be evaluated. The proposed method using these data is expected to be able to contribute to more quantitative risk assessment, and thus more rigorous safety analysis of various process facilities and devices under changing operating conditions.

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