Korean J. Chem. Eng., **30**(4), 813-822 (2013)
DOI: 10.1007/s11814-012-0201-2

Free convective flow over a vertical plate in a doubly stratified medium with electrophoresis, heat source/sink and chemical reaction effects

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(*Received 14 August 2012 • accepted 16 November 2012*)

Abstract−We analyzed the problem of unsteady, incompressible free convective doubly stratified flow past a semi-infinite vertical plate with the influence of electrophoresis, heat source/sink and chemical reaction. The partial differential equations governing the flow are solved by employing an implicit finite difference scheme of Crank-Nicolson type. The effect of heat generation and absorption in stratified and unstratified flow are examined and hence the influence of stratification on velocity, temperature and concentration are investigated and presented graphically. Further, the impact of the electrophoresis on particle concentration in the presence of generative and destructive reaction is analyzed. As well, the effects of the physical parameters on local and average values of skin friction, Nusselt number and Sherwood number are also investigated and illustrated graphically. The particular solutions of the present results are compared with the existing solution in literature and are found to be in good agreement.

Key words: Stratification, Electrophoresis, Heat Source, Heat Sink, Chemical Reaction

INTRODUCTION

The problem of natural convective flow caused by combined buoyancy effects of thermal and mass diffusion has been a subject of interest for years and analyzed by several authors. The phenomenon of heat and mass transfer in free convective flow finds a large number of applications in various industries and environments such as polymer production, chemical catalytic reactors, electrolytic processes, food processing, nuclear power plants and geophysical flows. The problem of transient free convective flow past a semi-infinite isothermal vertical plate was first explored by Siegel [1] using an integral method. Gebhart and Pera [2] analyzed the vertical natural convection flows resulting from the combined buoyancy effects of thermal and mass diffusion. Hellums and Churchill [3], Callahan and Marner [4] studied the problem of transient, free convective flow past a vertical plate using an explicit finite difference scheme, while Soundalgekar and Ganesan [5] used an implicit finite difference scheme of Crank-Nicolson type to examine the free convection problem. Later, Rani and Kim [6] implemented the implicit finite difference scheme to study the Dufour and Soret effects in unsteady natural convection flow.

Furthermore, the effect of stratification is also a vital characteristic that has to be considered in the analysis of heat and mass transfer. Stratification of fluids occurs because of either temperature variations or concentration differences or due to the presence of different fluids of different densities. Notion of stratification is also important in still water like lakes, ponds etc. It is of great importance to monitor the temperature stratification and concentration differences of hydrogen and oxygen in such cases as it may directly affect the growth rate of all cultured species. As well, the concept of stratification is crucial in electrochemical processes and reactors. In case of lead acid batteries, the stratification of temperature and concentration causes internal leakage and sedimentation at the bottom of the battery, which in turn reduces the life of the battery and its capacity. Similarly, in the case of pressurized water reactors, significant heat transfer effects occur when the corium and metallic components stratify, which may lead to severe accidents. Also, thermal stratification occurs under reactor scram condition. In view of the importance of stratification at various circumstances, authors have explored the influence of stratification in the heat and mass transfer study. Yet, very few studies investigate the combined effects of thermal and mass stratification in a free convective flow. Chen and Eichhorn [7] explored the natural convection flow over a heated vertical surface in a thermally stratified medium by using local non-similarity technique. Yang et al. [8] have shown that the similarity solution is not possible for natural convection from an isothermal vertical plate to a stable thermally stratified ambient, and hence more complex cases can be handled by finite-difference solution of the double diffusive free convection problem. Jaluria [9] employed finite difference scheme to study the vertical, two-dimensional natural convection flow over a heated vertical surface, the ambient medium being a stably stratified due to temperature increase with height. It was determined that for the flow adjacent to a vertical plate, the velocity and buoyancy levels were found to be lowered due to stratification. Later, Angirasa and Srinivasan [10,11] investigated the combined heat and mass transfer effects in a thermally stratified medium using an explicit finite difference method. Saha and Hossain [12] studied combined buoyancy effects due to thermal and mass diffusion in a thermally stratified media. Although the above proposed studies analyze the effects in a thermally stratified medium, very few investigations have been done to study the influence of thermal and mass stratification on heat and mass transfer in a natural convective flow. Rathish Kumar and Shalini Gupta [13] examined the problem of non-Darcian

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combined heat and mass transfer from a vertical wavy wall in a doubly stratified porous medium and solved numerically by local non-similarity technique. Srinivasacharya and RamReddy [14] analyzed the steady, free convective problem of heat and mass transfer in a doubly stratified non-Darcy micropolar fluid using Keller-Box method.

As monitoring stratification is important in reactors, it is also equally essential to monitor the particle concentration and control the level of ionic species in the reactor coolant as it can impact fuel performance and cause fouling of catalysts. This control of ionic species can be achieved by electrophoresis technique. Electrophoresis is a separations technique based on the mobility of ions in an electric field. The technique has received widespread attention in controlling particle transport path or preventing particle deposition, separation of ionic species and separation of charged biochemicals based on molecular properties etc. However, very few studies have been done to analyze the electrophoretic influence in the study of heat and mass transfer processes. Opiolka et al. [15] experimentally and numerically investigated thermophoretic and electrophoretic effects of particle deposition on horizontal flat surfaces. Further, Tsai et al. [16] proposed a theoretical study of aerosol particles to investigate the combined effect of thermophoresis and electrophoresis onto an axisymmetric wafer using similarity analysis. Recently, Tsai and Huang [17] examined the same in detail, in a mixed convective flow over a vertical flat plate through a porous medium. Hence, due to the importance of electrophoretic and stratification effects in the analysis of separation of chemicals or in bio-chemical processes and in reactors, an attempt is made to investigate the effect of electrophoresis on the transport of the fluid particles, particle concentration and diffusion in a doubly stratified medium.

Moreover, in practical applications such as electrochemical processes and reactors, occurrence of chemical reaction is anticipated the most. Particularly, the effect of electrophoresis with chemical reaction can be applied in the analysis of control of aerosol particles in an environment that may otherwise lead to great climatic changes resulting in natural disaster. The presence of foreign mass in the fluid causes some kind of chemical reaction, such as exothermic or endothermic, which may alter the diffusion rate tremendously. Also, in well mixed system, depending upon whether the reaction takes place at an interface or in the solution, the reaction is classified as homogeneous and heterogeneous, respectively. Chambre and Young [18] analyzed the problem of first-order destructive and generative chemical reactions in the neighbourhood of a flat plate. Andersson et al. [19] studied the diffusion of a chemically reactive species from a stretching sheet. Later, Muthucumaraswamy and Ganesan [20] studied the first-order chemical reaction on flow past an impulsively started vertical plate with uniform heat and mass flux. Palani and Kim [21] examined the first-order homogeneous chemical reaction in an unsteady free convective flow past a semi-infinite vertical plate.

It is realistic to consider the heating effect as it is inevitable in processes that involve electrophoresis and chemical reaction. In case of electrophoresis, heating effect is caused by the electroresistivity of the media and this could modify the temperature differences inside the boundary layer, which in turn would influence the driving force for hydrodynamics of the system. Further, the study of heat generation or absorption effects is important in fluids undergoing such exothermic or endothermic chemical reactions. Thus, in many industrial processes involving heat and mass transfer processes, the diffusing species can be generated or absorbed due to certain kind of chemical reaction with the ambient fluid, which can greatly affect the flow and hence the properties and quality of the final product. Vajravelu and Rollins [22] investigated heat transfer characteristics in an electrically conducting fluid over a stretched sheet with internal heat generation or absorption. Cheng [23] studied the problem of unsteady flow and heat transfer in the laminar boundary layer on a linearly accelerating surface with suction or blowing in the absence as well as presence of a heat source or sink is considered. Shanker [24] studied the radiation and mass transfer effects on unsteady MHD free convective fluid flow embedded in a porous medium with heat generation/absorption. Kandasamy et al. [25] used group theory transformation to study the problem of soret and dufour effect on free convective flow with thermophoresis and chemical reaction in the presence of heat source/sink.

Due to the considerable importance of the factors discussed, our objective is to study free convective doubly stratified flow over a vertical plate with the influence of electrophoresis, heat source/sink and chemical reaction. The present analysis may find its application in new emerging problems in reactors, electrochemical and biochemical processes. In the present study, the influence of heat source/sink in stratified and unstratified flow, effects of electrophoretic parameter in a generative as well in a destructive chemical reaction on velocity, temperature and concentration are investigated and presented graphically. As well, the local and average values of skin friction, Nusselt number and Sherwood number are also analyzed. The results are compared with relevant results in the existing literature and are found to be in good agreement.

MATHEMATICAL ANALYSIS

A two-dimensional transient, laminar free convective flow of a viscous incompressible fluid past a semi-infinite vertical plate in a doubly stratified medium is considered. Initially at time $t'=0$ the fluid and the plate are assumed to be at the same temperature and concentration. At time $t'>0$ the temperature of the plate is changed to T_w and the concentration level near the plate is assumed to C_w. In the ambient, the temperature and concentration of the stratified medium increases linearly with height, where $T_{\infty,0}$ and $C_{\infty,0}$ being its value at x=0 respectively. All the fluid physical properties are assumed to be constant except for the body force terms. It is assumed that fluid is a heat absorbing/generating fluid and there exists a first-order chemical reaction between the diffusing species and the fluid. The factors that affect species transport include convection, Brownian diffusion, turbulence, sedimentation, inertial effect, thermophoresis, electrophoresis and surface geometry. The effects of electrophoresis and convection are taken into account to understand the particle transport and diffusion mechanism. The *x*-axis is taken along the plate in the vertically upward direction and the *y*-axis is taken normal to the plate as shown in Fig. 1.

Under the above assumptions, the governing boundary layer equations for the flow with usual Boussinesq's approximation (Herrmann Schlichting [26]) are as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

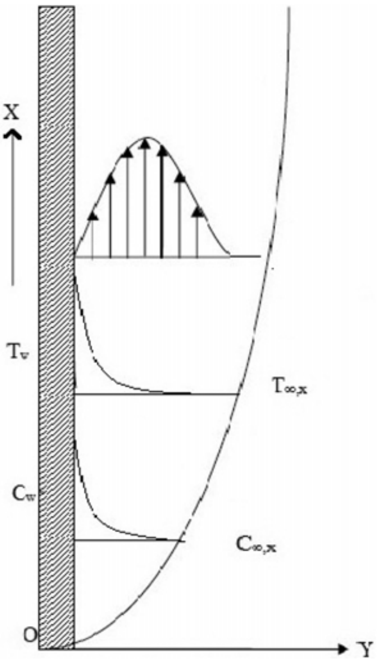

Fig. 1. Schematic diagram.

$$\frac{\partial u}{\partial t'}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}+g\beta_T(T'-T_{\infty,x})+g\beta_C(C'-C_{\infty,x}) \tag{2}$$

$$\frac{\partial T'}{\partial t'}+u\frac{\partial T'}{\partial x}+v\frac{\partial T'}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{Q'(T'-T_{\infty,x})}{\rho c_p} \tag{3}$$

$$\frac{\partial C'}{\partial t'}+u\frac{\partial C'}{\partial x}+v\frac{\partial C'}{\partial y}=D\frac{\partial^2 C'}{\partial y^2}-\frac{\partial(C'v_E)}{\partial y}-K'_C(C'-C_{\infty,x}) \tag{4}$$

where u and v are velocity component in x and y-direction, respectively, T' is the temperature and C' is the particle concentration, g is the acceleration due to gravity, ρ is the density, α and D are the effective thermal and solutal diffusivities of the medium, β_T is the volumetric thermal expansion coefficient, β_C is the volumetric coefficient of expansion with concentration, Q' is the heat source/sink coefficient, c_p is the specific heat at constant pressure K'_C, is the chemical reaction parameter, $T_{\infty,x}$, $C_{\infty,x}$ are ambient temperature and concentration, which is a linear function of x, v_E is the electrophoresis velocity that aids in investigating the impact of the electrophoresis on the mass transfer processes. Under boundary layer approximations the concentration gradient in the y-direction is very much larger than in the x-direction, and therefore only the electrophoretic velocity in the y-direction is considered.

The initial and boundary conditions are as follows:

$t'\leq 0$, u=0, v=0, $T'=T_{\infty,x}$, $C'=C_{\infty,x}$
$t'>0$, u=0, v=0, $T'=T_w$, $C'=C_w$ at y=0
u=0, v=0, $T'=T_{\infty,0}$, $C'=C_{\infty,0}$ at x=0
$u\rightarrow 0$, $T'\rightarrow T_{\infty,x}$, $C'\rightarrow C_{\infty,x}$ as $y\rightarrow\infty$ (5)

The non-dimensional quantities are defined as follows:

$$X=\frac{x}{L};\ Y=\frac{y}{L}Gr^{\frac{1}{4}};\ U=\frac{uL}{\upsilon}Gr^{\frac{-1}{2}};\ V=\frac{vL}{\upsilon}Gr^{\frac{-1}{4}};\ T=\frac{(T'-T_{\infty,x})}{(T_w-T_{\infty,0})};$$

$$C=\frac{(C'-C_{\infty,x})}{(C_w-C_{\infty,0})};\ t=\frac{t'\upsilon}{L^2}Gr^{\frac{1}{2}};\quad Gr=\frac{g\beta_T L^3(T_w-T_{\infty,0})}{\upsilon^2};\ N=\frac{\beta_C(C_w-C_{\infty,0})}{\beta_T(T_w-T_{\infty,0})};$$

$$S_T=\frac{\frac{dT_{\infty,x}}{dX}}{(T_w-T_{\infty,0})};\ S_M=\frac{\frac{dC_{\infty,x}}{dX}}{(C_w-C_{\infty,0})};\ \alpha=\frac{k}{\rho c_p};\ Sc=\frac{\upsilon}{D};\ Pr=\frac{\mu c_p}{k};$$

$$Q=\frac{Q'Gr^{\frac{-1}{2}}L^2}{\rho c_p\upsilon};\ V_E=\frac{v_E L}{\upsilon}Gr^{\frac{-1}{4}};\ K_C=\frac{K'_C L^2}{\upsilon}Gr^{\frac{-1}{2}};$$

The non-dimensional form of Eqs. (1), (2), (3) and (4) are obtained as

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{6}$$

$$\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=\frac{\partial^2 U}{\partial Y^2}+T+NC \tag{7}$$

$$\frac{\partial T}{\partial t}+U\frac{\partial T}{\partial X}+V\frac{\partial T}{\partial Y}=\frac{1}{Pr}\frac{\partial^2 T}{\partial Y^2}-S_T U+QT \tag{8}$$

$$\frac{\partial C}{\partial t}+U\frac{\partial C}{\partial X}+V\frac{\partial C}{\partial Y}=\frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2}-S_M U-V_E\frac{\partial C}{\partial Y}-K_C C \tag{9}$$

where N is the buoyancy ratio parameter, Pr is the Prandtl number, Q is the heat source/sink parameter, Sc is the Schmidt number, S_T and S_M are the thermal and mass stratification parameter, respectively, K_c is the chemical reaction parameter, V_E is the electrophoretic parameter.

Thus, the boundary conditions (5) reduce as,

$t\leq 0$, U=0, V=0, T=0, C=0
$t>0$, U=0, V=0, $T=1-S_T X$, $C=1-S_M X$ at Y=0
U=0, V=0, T=0, C=0 at X=0
$U\rightarrow 0$, $T\rightarrow 0$, $C\rightarrow 0$ as $Y\rightarrow\infty$ (10)

NUMERICAL PROCEDURE

The two-dimensional, non-linear, coupled partial differential Eqs. (6)-(9) under the initial and boundary conditions (10) are solved using implicit finite difference scheme of Crank-Nicolson type which

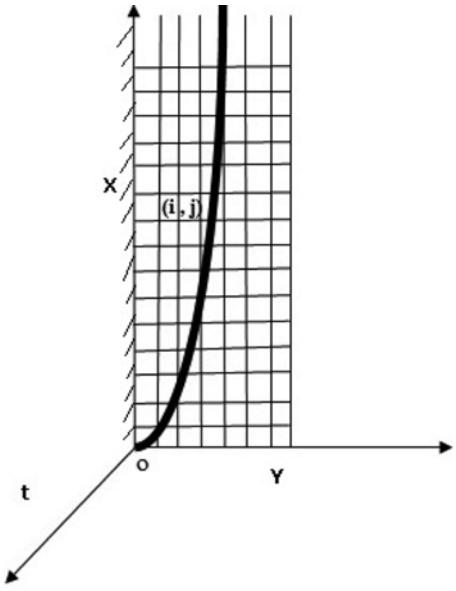

Fig. 2. Discretization of domain.

converges faster and is also unconditionally stable.

The region of integration is considered as a rectangle with sides $X_{max}=1$ and $Y_{max}=14$, where Y_{max} corresponds to $Y=\infty$. The Y_{max} was chosen as 14 after some preliminary investigations so that the last two of the boundary conditions of (10) are satisfied. The domain is divided into grids with mesh sizes $\Delta X=0.05$, $\Delta Y=0.25$ and $\Delta t=0.01$ in X-, Y- and t-direction, respectively, as shown in Fig. 2. The dependent variable, say temperature, at the intersecting point is represented by the grid node $T_{i,j}^{k}$ where the subscript i and j assigns the grid point along X- and Y-direction, while superscript k along t direction.

The finite difference equations at every internal nodal point for a particular i-level constitute a tridiagonal system. This system is solved by applying the Thomas algorithm as described in Carnahan et al. [27]. Thus the values of C, T, U, V are known at all nodal points in the rectangular region at $(k+1)^{th}$ time level by repeating the procedure for various i-levels. Computations are carried out for all the time levels until the steady state is reached. The steady state solution is assumed to have been reached, when the absolute difference between the values of U, as well as temperature T and concentration C at two consecutive time steps are less than 10^{-5} at all grid points.

The scheme proves to be unconditionally stable using von Neumann technique, as shown by Soundalgekar and Ganesan [5]. The local truncation error is $O(\Delta t^2+\Delta X+\Delta Y^2)$ and it tends to zero as Δt, ΔX and ΔY tend to zero. Hence the scheme is compatible. Stability and compatibility ensures the convergence of the scheme.

SKIN FRICTION, NUSSELT NUMBER AND SHERWOOD NUMBER

The local as well as average values of skin friction, Nusselt number and Sherwood number in dimensionless form are as follows:

The non-dimensional forms of local skin friction, Nusselt number and Sherwood number are obtained as

$$\tau_X = Gr^{\frac{3}{4}} \frac{\partial U}{\partial Y}\bigg|_{Y=0} \tag{11}$$

$$Nu_X = -Gr^{\frac{1}{4}} \frac{X \frac{\partial T}{\partial Y}\Big|_{Y=0}}{1-S_T X} \tag{12}$$

$$Sh_X = -Gr^{\frac{1}{4}} X \frac{\frac{\partial C}{\partial Y}\Big|_{Y=0}}{(1-S_M X)} \tag{13}$$

The non-dimensional forms of average skin friction, Nusselt number and Sherwood number are obtained as

$$\bar{\tau}_X = Gr^{\frac{3}{4}} \int_0^1 \frac{\partial U}{\partial Y}\bigg|_{Y=0} dX \tag{14}$$

$$\overline{Nu}_X = -Gr^{\frac{1}{4}} \int_0^1 \frac{\frac{\partial T}{\partial Y}\Big|_{Y=0}}{(1-S_T X)} dX \tag{15}$$

$$\overline{Sh}_X = -Gr^{\frac{1}{4}} \int_0^1 \frac{\frac{\partial C}{\partial Y}\Big|_{Y=0}}{(1-S_M X)} dX \tag{16}$$

The derivatives involved in Eqs. (11)-(16) are evaluated by using a five-point approximation formula and the integrals are evaluated by Newton-Cotes closed integration formula.

RESULTS AND DISCUSSION

The numerical results are presented with the help of graphical illustrations to acquire a clear vision of the physical problem. In the current investigation, the influence of the electrophoretic parameter V_E is analyzed for values between 0 and 1. As well, the effect of the thermal stratification level and mass stratification level has been analyzed on the double diffusive natural convection process for a range of thermal stratification and mass stratification, $0 \le S_T \le 1$ and $0 \le S_M \le 1$ respectively (Kumar and Gupta [13]). The Prandtl number is preferred to be 0.73 for air. The Schmidt number value is chosen to be 0.2 (Hydrogen) that represents physically buoyant gases diffusing in a boundary layer convection flow at a temperature of 25 ℃ and one atmospheric pressure. The velocity, temperature and concentration of the fluid flow are shown graphically to observe the impact of physical parameters.

To check the accuracy of the result, the particular solution of the present result is compared with the published result in the literature. The velocity profiles for $Pr=0.7$, $Sc=0.94$, $N=1.0$, $V_E=0$, $K_C=0$, $Q=0$ of the present result are compared with the existing solution of Gebhart and Pera [2] in the absence of doubly stratified porous medium. The comparison of the same is depicted in Fig. 3. Similarly, the temperature and concentration profiles for $Pr=0.7$, $Sc=0.7$, $N=2$, $S_T=0.004$, $S_M=0$, $Q=0$, $K_C=0$, $V_E=0$ of the present result are compared with the existing solution of Srinivasan and Angirasa [11] as depicted in Fig. 4. From Fig. 3 and Fig. 4, the present results are in good agreement with the existing solution.

Figs. 5(a), (b)-7(a), (b) depict the effect of heat source/sink parameter Q on velocity, temperature and concentration profiles in doubly stratified and unstratified flow. Here, while $Q>0$ represents heat generation, $Q<0$ represents heat absorption, i.e., addition and removal of thermal energy to the flow, respectively. Fig. 5(a), (b) illustrate that the generation of heat ($Q>0$) raises the velocity initially and this is sustained for a considerable distance from the plate towards the free stream and after a certain state a flow reversal (reversal of

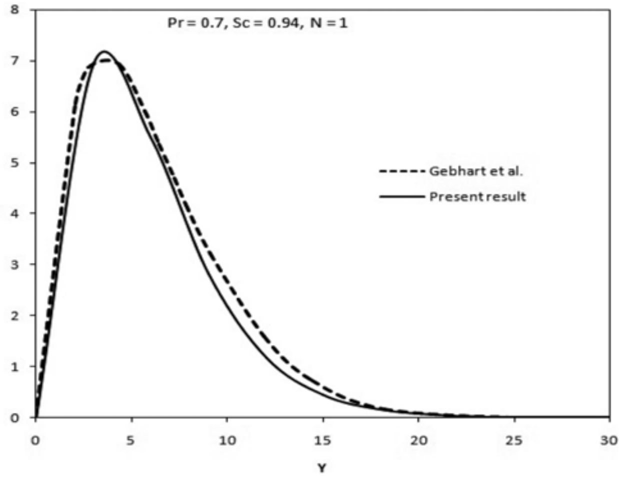

Fig. 3. Comparison of velocity profiles with Gebhart [2].

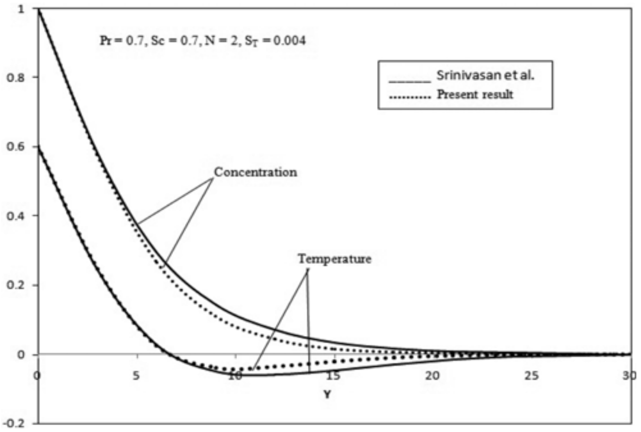

Fig. 4. Comparison of temperature and concentration profiles with Srinivasan and Angirasa [11].

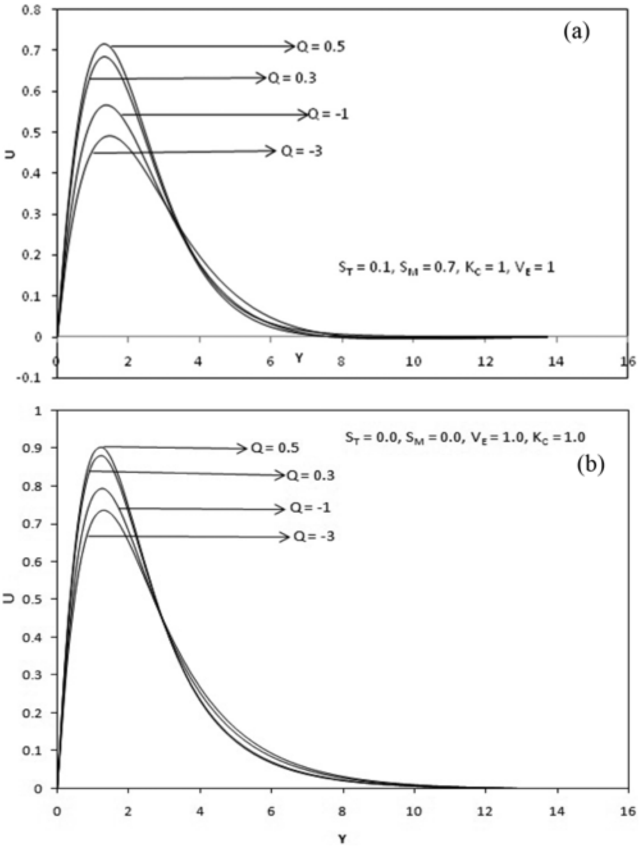

Fig. 5. (a) Heat source/sink effects on velocity in stratified medium at X=1.0. (b) Heat source/sink effects on velocity in unstratified medium at X=1.0.

flow behavior) occurs. As well, Fig. 6(a), (b) and 7(a), (b) describe that the heat generation enhances the temperature and reduces the concentration profiles, respectively. Physically, this is because an increase in the heat source raises the temperature of the fluid, which onsets a rise in velocity. Subsequently, due to the existence of small thermal gradient towards the free stream, a deceleration in velocity follows, and hence an occurrence of flow reversal is observed. Also, it is noticed that as heat sinks (Q<0), temperature decreases and the concentration increases.

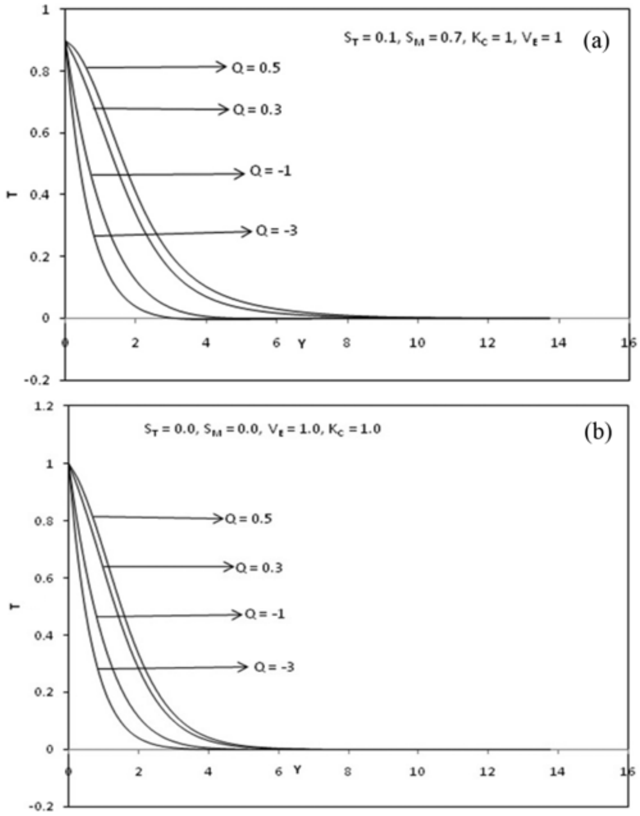

Fig. 6. (a) Heat source/sink effects on temperature in stratified medium at X=1.0. (b) Heat source/sink effects on temperature in unstratified medium at X=1.0.

Furthermore, Figs. 5(a), (b)-7(a), (b) illustrate that the velocity, temperature and concentration profiles drop off in stratified flows than in unstratified flows. Physically, the rise in thermal and mass stratification increases the ambient temperature and concentration, respectively, linearly with the height. As an aiding buoyancy flow has been considered, the reduction in the temperature and concentration gradient reduces the thermal and concentration driven buoyancy, which decelerates the velocity of the flow. Though temperature decreases in stratified flows when compared with unstratified case, it is particular to note from Fig. 6(a), (b) that generation of heat (Q>0) enhances the temperature significantly in stratified flow than in unstratified flow. As well, Fig. 7(a), (b) describes that in unstratified flow, heat source/sink does not have a significant effect on concentration profiles whereas in stratified flow the concentration drops off with an increment in the heat source/sink parameter. This is because, as observed already, the velocity is reduced in a stratified medium than for unstratified medium. The reduction in velocity causes diffusion to dominate over convection. Hence, a significant effect on the concentration profile is noted in a stratified medium than unstratified medium. Besides, a negative concentration profile has been observed. The fact that nondimensional concentration attains a negative value in stratified flow is because for a high mass stratification, the concentration at the wall is reduced to a value lower than the ambient and hence leads to a negative value. A similar result was already obtained by Murthy et al. [28] for steady case, who re-

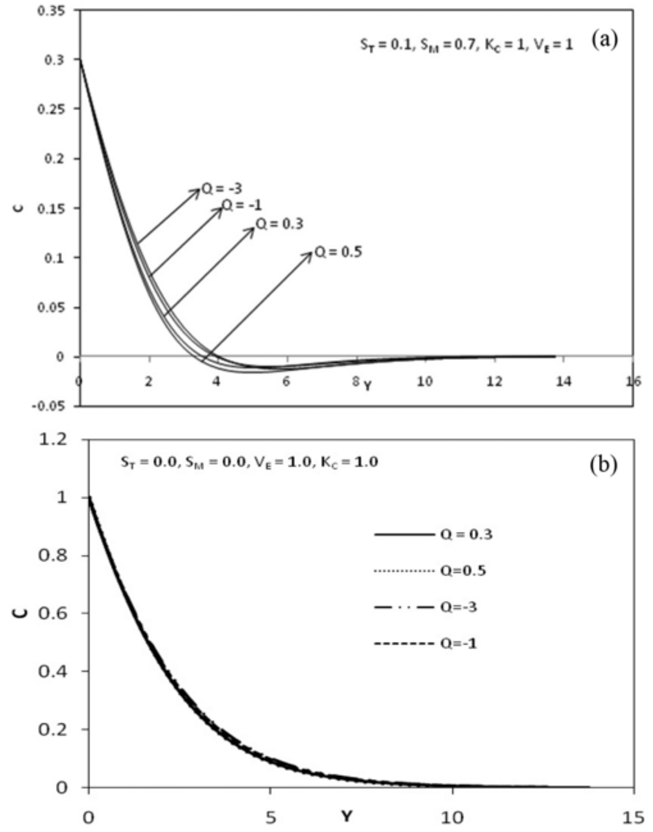

Fig. 7. (a) Heat source/sink effects on concentration in stratified medium at X=1.0. (b) Heat source/sink effects on concentration in unstratified medium at X=1.0.

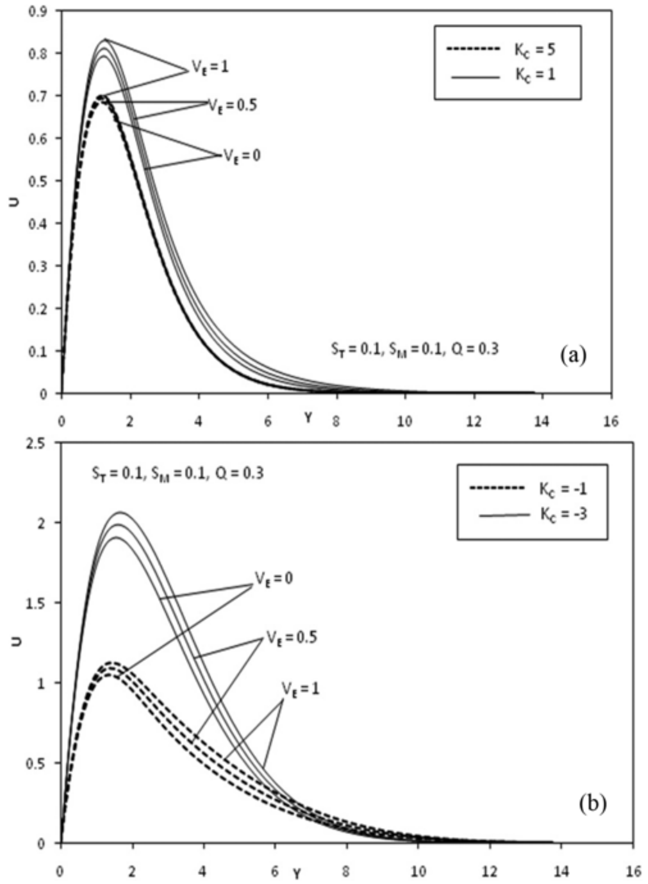

Fig. 8. (a) Electrophoretic parameter effects on velocity in destructive reaction at X=1.0. (b) Electrophoretic parameter effects on velocity in generative reaction at X=1.0.

ported that the temperature and concentration become negative in the boundary layer depending on the relative intensity of the thermal and solutal stratification.

Figs. 8(a), (b), 9(a), (b) and 10(a), (b) display the effect of electrophoretic parameter on velocity, temperature and concentration profiles in doubly stratified medium with generative and destructive chemical reaction. The figures show that in generative and destructive chemical reaction, an increase in the electrophoretic parameter elevates the velocity, particle concentration and decreases the temperature profiles. Increment in the velocity of the fluid enhances the particle velocity, and this is apparent as an increase in the electrophoretic parameter represents a high electrophoretic force, which in turn accelerates the velocity of the fluid particles. Similarly, 9(a), (b) illustrate that a high electrophoretic force decreases the temperature, and as a result, the particles migrate towards the surface with an increase in the particle concentration near the surface as shown in 10(a), (b).

In the present work, a homogeneous first order chemical reaction is considered; hence the reaction parameter K_c has the dimension of the reciprocal of time. The diffusion species either can be destroyed or generated in a homogeneous reaction, and hence Eq. (9) can be adjusted to meet these circumstances if one takes $K_c<0$ for generative reaction and $K_c>0$ for destructive reaction. For generative and destructive chemical reaction ($K_c<0$ and $K_c>0$), a high chemical reaction parameter K_c, decreases the velocity, concentration profiles while rises the temperature profiles. However, Figs. 8(b) and 9(b) depict that in a generative chemical reaction, as chemical reaction parameter K_c increases, the velocity decreases and temperature increases up to a certain distance from the wall, and then behavior of the flow velocity and temperature is reversed. Further, the temperature attains a negative value in both the generative and destructive chemical reaction. This is because the presence of thermal stratification increases the ambient temperature with height. At a fixed vertical location, the fluid coming from below is lower in value than the surface or ambient temperature. Thus, a negative temperature arises downstream, and this defect increases with an increase in S_T. These characteristics have been observed and described earlier by Yang et al. [8]. As noted from Fig. 10(a), (b), though increase in the chemical reaction parameter decreases the concentration in both the destructive and generative reaction, there is an increase in the concentration in a generative reaction compared to a destructive reaction. In a generative reaction, diffusion species are generated, which increases the concentration, while in a destructive reaction, the diffusion species are destroyed, which in turn reduces the concentration. Also, Figs. 8(a), 9(a) and 10(a) show that in a destructive reaction the effect of electrophoretic parameter on velocity, temperature and concentration decreases as the chemical reaction parameter increases.

Figs. 11-16 show that in the presence of stratification, for a high electrophoretic parameter, the wall shear stress and rate of heat trans-

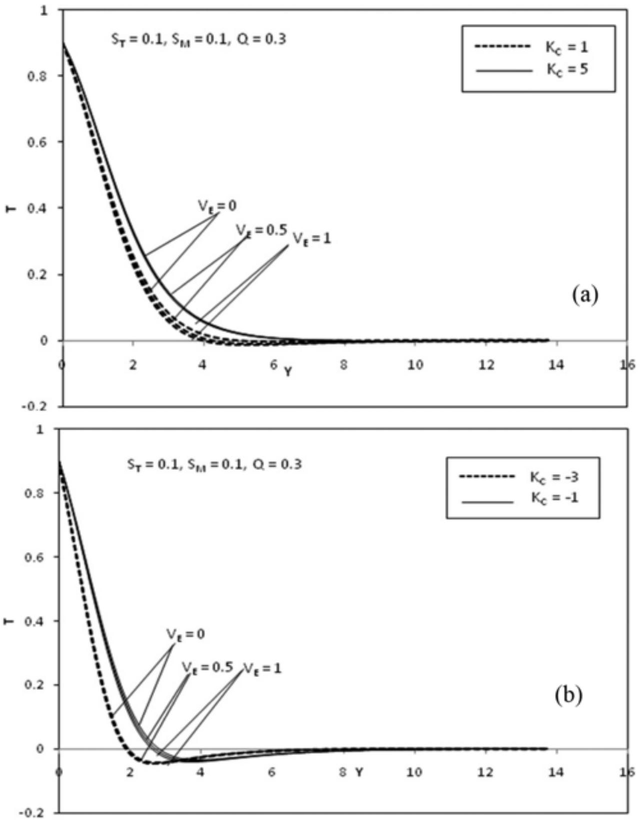

Fig. 9. (a) Electrophoretic parameter effects on temperature in destructive reaction at X=1.0. (b) Electrophoretic parameter effects on temperature in generative reaction at X=1.0.

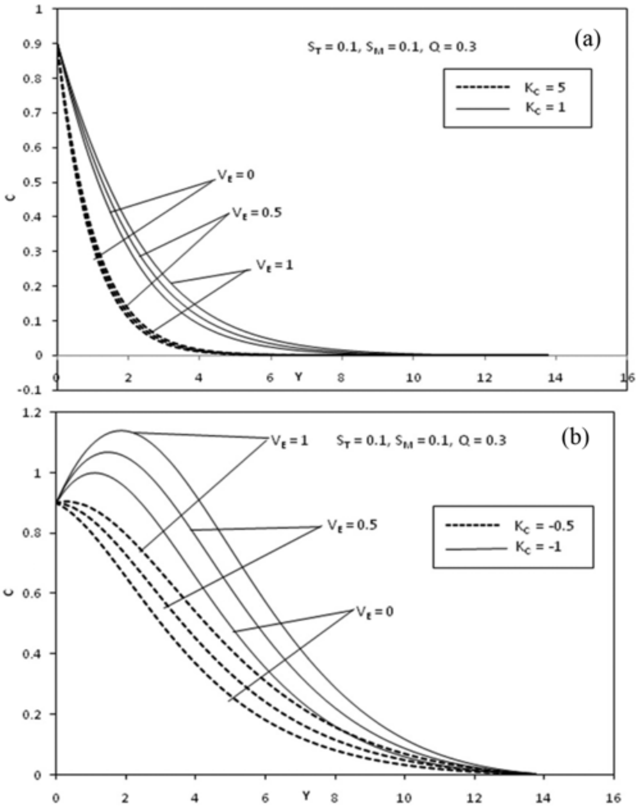

Fig. 10. (a) Electrophoretic parameter effects on concentration in destructive reaction at X=1.0. (b) Electrophoretic parameter effects on concentration in generative reaction at X=1.0.

fer increase, while the rate of mass transfer decreases. Physically, it is apparent as an increase in the electrophoretic force accelerates the velocity of the fluid, which in turn improves the shear stress along the wall. High electrophoretic parameter causes an increase in the particle concentration. As a result, the particle concentration gradient between the wall and the ambient is reduced, and hence the rate of transfer of species or particles retards.

Figs. 11-16 also depict the effect of the heat source/sink parameter on skin friction, Nusselt and Sherwood number. As discussed earlier, an increase in the source/sink parameter Q increases both the velocity and temperature profiles as seen from Figs. 5(a) and 6(a). The increase in velocity increases the frictional force along the wall, which in turn increases the skin friction; whereas an increase in temperature decreases the temperature gradient and hence decreases the rate of heat transfer. It is observed from Fig. 11(a) that the influence of heat source/sink parameter Q on wall shear stress is significant compared to electrophoresis parameter V_E. This is clear from Figs. 5(a) and 8, as the effect of heat source/sink on velocity is more considerable than the influence of electrophoresis on velocity. Also, note that from Fig. 13(a), as the heat source/sink parameter increases, the effect of V_E on local Nusselt number increases. Fig. 15(a) determines that the rate of mass transfer improves with an increase in the heat source/sink parameter Q. As noticed and discussed from Fig. 7(a), an increase in the heat source/sink parameter Q decreases the concentration profiles. This in turn increases the concentration gradient and results in an increment in the rate of mass transfer.

From Figs. 11(b) and 12, an increase in the chemical reaction parameter K_c decreases the skin friction. As shown earlier from Figs. 8(a) and 8(b), an increase in the chemical reaction parameter K_c decreases the velocity of the flow. As a result, the drag between the wall and the flow decreases, which in turn reduces the skin friction. Also from Figs. 8(a) and 8(b), influence of V_E on velocity is substantial in generative reaction compared with destructive reaction. Thus, an increase in the velocity of the flow augments the drag between the surface and fluid in a generative than in destructive reaction. As a result, the effect of V_E on local skin friction is significant in generative reaction than destructive reaction, and clearly demonstrated from Fig. 11(b). Figs. 15(b) and (16) show that an increase in the chemical reaction parameter K_c increases the rate of mass transfer. As concentration of species is reduced with an increase in K_c observed from Figs. 10(a) and 10(b), the concentration gradient increases which in turn increases the species diffusion rate. It is also observed that the local Sherwood number decreases significantly in generative chemical reaction and attains a negative value. Fig. 13(b) suggests that the rate of heat transfer is hampered with an increment in the chemical reaction parameter K_c. From Fig. 9(a) and 9(b) an increase in chemical reaction parameter increases the temperature. This decreases the temperature gradient and hence decreases the heat transfer rate. However, from Fig. 14, the influence of the chemical reaction parameter is not that significant on average Nusselt number.

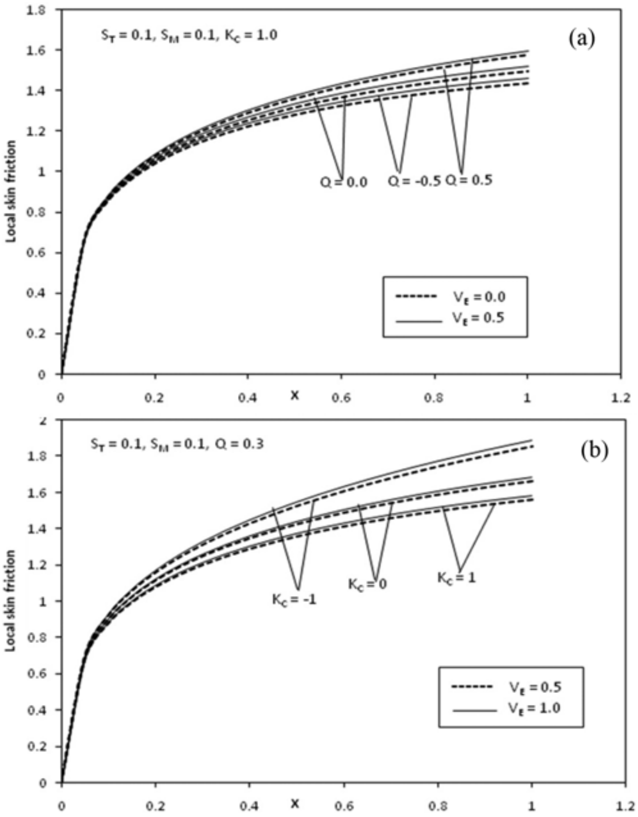

Fig. 11. (a) Heat source/sink effects on local skin friction at X=1.0. (b) Chemical reaction effects on local skin friction at X=1.0.

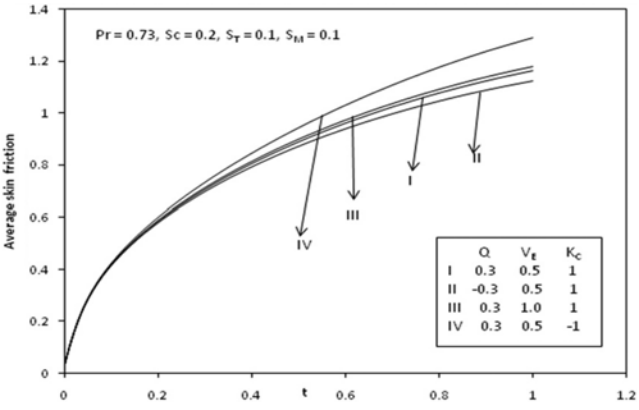

Fig. 12. Effects of Q, V_E and K_C on average skin friction.

CONCLUSION

We investigated the influence of chemical reaction, heat source/sink and electrophoretic effects in a free convective, transient, incompressible flow over a vertical plate with a doubly stratified ambient. Implicit finite difference scheme of Crank-Nicolson type was employed to obtain the solution of the governing equation. The particular solutions reported in this paper were validated by comparing with the solutions existing in the previously published paper. Our results show a good agreement with the existing work in the literature. The results are summarized as follows:

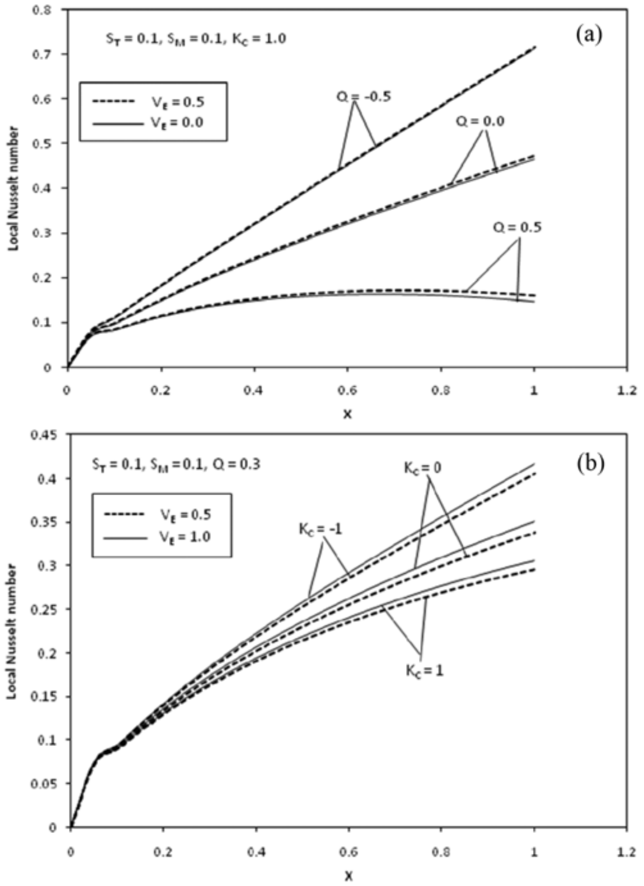

Fig. 13. (a) Heat source/sink effects on local Nusselt number. (b) Chemical reaction effects on local Nusselt number.

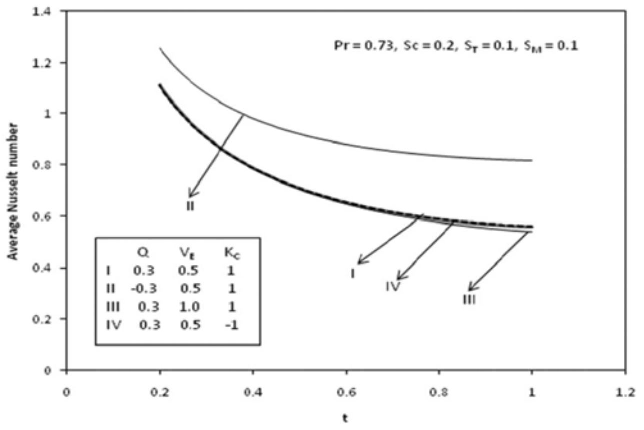

Fig. 14. Effects of Q, V_E and K_C on average Nusselt number.

1. Velocity, temperature and concentration drop in doubly stratified medium when compared to unstratified flow.

2. Heat generation elevates the temperature and reduces the concentration.

3. High electrophoretic parameter enhances velocity, concentration and lowers the temperature.

4. For both generative and destructive chemical reaction, there is a drop in velocity, concentration while a rise in temperature.

5. In destructive reaction, the influence of electrophoresis on veloc-

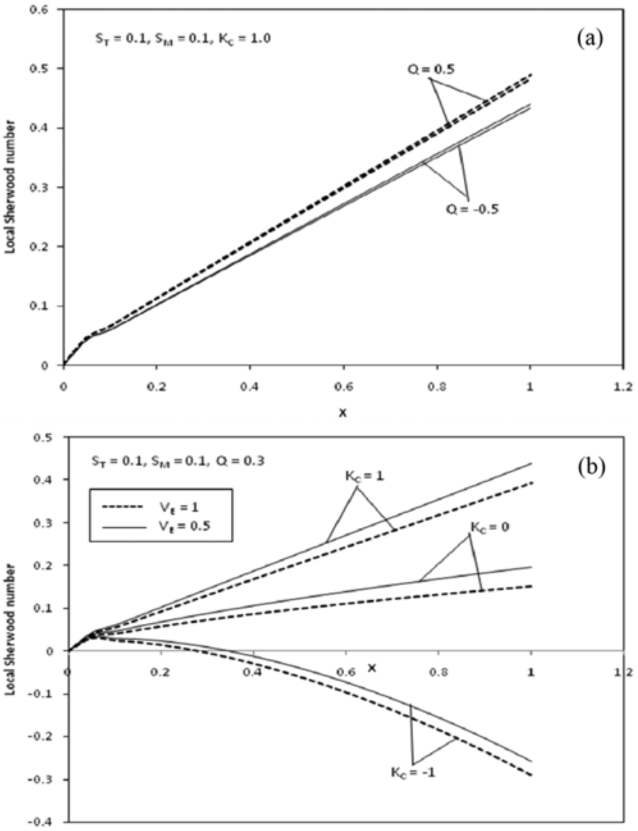

Fig. 15. (a) Heat source/sink effects on local Sherwood number. (b) Chemical reaction effects on local Sherwood number.

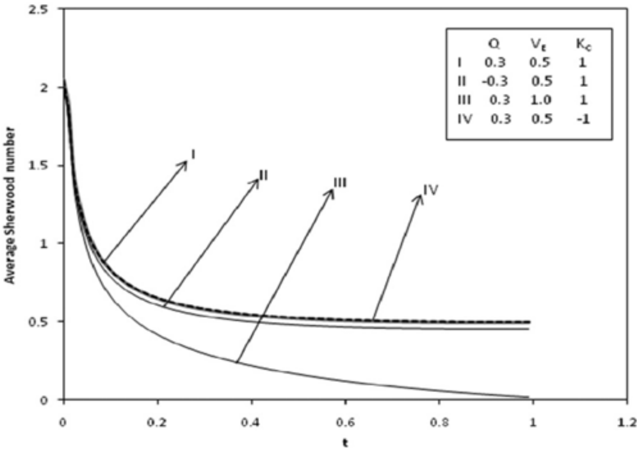

Fig. 16. Effects of Q, V_E and K_C on average Sherwood number.

ity, temperature and concentration declines as the chemical reaction parameter increases.

6. Negative temperature and negative concentration values observed are due to the influence of thermal and mass stratification.

7. Generation of heat enhances the temperature significantly in stratified flow compared with unstratified flow.

8. Increment in electrophoretic parameter increases the wall shear stress and rate of heat transfer while decreases the rate of mass transfer.

9. Electrophoretic effect on wall shear stress is more significant in generative reaction than in destructive reaction.

NOMENCLATURE

b : variable mass diffusivity parameter
C' : concentration of the fluid
C : dimensionless concentration
D : mass diffusion coefficient
Gr : thermal Grashof number
k : thermal conductivity
K_C' : chemical reaction parameter
K_C : dimensionless chemical reaction parameter
N : buoyancy ratio parameter
Nu_X : local Nusselt number
$\overline{Nu}_X$: average Nusselt number
Pr : Prandtl number
Q' : heat source/sink coefficient
Q : dimensionless heat source/sink parameter
Sc : schmidt number
Sh_X : local Sherwood number
$\overline{Sh}_X$: average Sherwood number
S_T : thermal stratification parameter
S_M : mass stratification parameter
t' : time
t : dimensionless time
T' : temperature of the fluid
T : dimensionless temperature
u : velocity component along the plate
U : dimensionless velocity component along the X-direction
v : velocity component normal to the plate
V : dimensionless velocity component along Y-direction
v_E : electrophoretic velocity
V_E : dimensionless electrophoretic velocity
x : spatial coordinate along the plate
X : dimensionless spatial coordinate along the plate
y : spatial coordinate normal to the plate
Y : dimensionless spatial coordinate normal to the plate

Greek Symbols

α : thermal diffusivity
β_T : volumetric thermal expansion coefficient
β_C : volumetric coefficient of expansion with concentration
ρ : density
ν : kinematic viscosity
τ_X : local skin friction
$\overline{\tau}_X$: average skin friction

Subscripts

w : conditions on the wall
∞ : conditions in the free stream

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