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# The application of chattering-free sliding mode controller in coupled tank liquid-level control system

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**Abstract**−A chattering-free sliding mode controller (CFSMC) is proposed to realize level position control of liquid level system for two coupled water tanks, as is often encountered in practical process control. The controller is used due to its robustness against large parameter variation, disturbances rejection and reduction in chattering. Experimentation of the coupled tank system is realized in two different configurations: configuration 1 and configuration 2. In configuration 1, the water level in the top tank is controlled by a pump. In 2, the water level in the bottom tank is controlled by the water flow coming out of the top tank. The validity of the proposed controller is verified by means of a practical testing on an experimental liquid level control device.

Key words: Liquid Level Control System, Sliding Mode Control

## INTRODUCTION

The control of liquid level in tanks and flow between tanks is a basic problem in the process industries. The process industries require liquids to be pumped, stored in tanks and then pumped to another tank. To control liquid level the traditional method PID control is used due to its reliability, simple structure and easy parameters adjusts [1-3]. In the practical application of a level control system, an ordinary PID control is not sufficient for precise control. To perform high precision liquid level control and good tracking precision in the presence of the system nonlinearities and parameter uncertainties, it is needed to use nonlinear control method to solve these problems effectively and achieve precise control. As a solution, a fuzzy controller is proposed to overcome nonlinearity, parameter variation and disturbances [4-7]. Many existing experiments have demonstrated that an adaptive fuzzy controller can be applied to the system whose dynamic model is not well defined or not available at all and has proven to be a strong tool for controlling nonlinear systems. However, when it comes to certain other situations, such as large delay, the control performance of the fuzzy controller is deteriorated. Also, neural network [8-12] and genetic algorithm [13-16] based controllers are proposed as effective tools for nonlinear controller design. Both controllers offer exciting advantages such as adaptive learning, fault tolerance, generalization and disadvantages, which are complex learning algorithm and computational requirement. The sliding mode control has gained more attraction for the machine control applications [12,15-18]. The SMC is a very effective approach for the solution of the problem due to its robustness to parameter variations, easy implementation, fast dynamic response, and disturbance rejection. A comprehensive review of the sliding mode control has been presented in [19,20]. It has been reported widely that sliding mode control exhibits unwanted motion, so-called chattering. This chattering is caused due to discontinuity of the control actions. To avoid this chattering, some solutions have been proposed in the literature [19-21].

In this paper, real time application of a chattering-free sliding mode control [20-22] is used for level control of experimental setup of liquid level system due to its properties such as robustness against large parameter variation and disturbances rejection. The coupled tank system is used in two different configurations: configuration 1 and configuration 2. Our study is focused on the level control of the top tank in configuration 1 and level control of bottom tank in configuration 2. The experimental results obtained prove that the

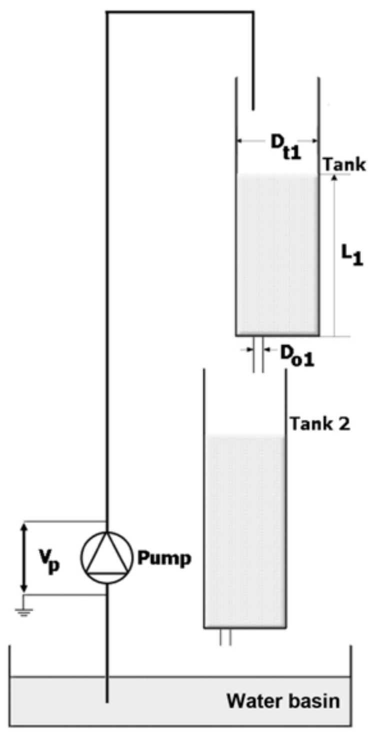


**Fig. 1. Tank configuration 1.**

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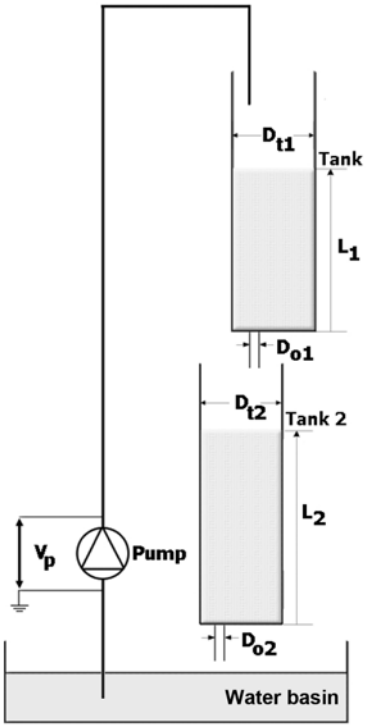


**Fig. 2. Tank configuration 2.**

controller is robust to liquid level changes as well as to disturbances and can also follow command trajectories without causing any chattering.

## MODELING OF THE COUPLED TANKS SYSTEM

### 1. Single Tank Model (Configuration 1)

Single tank system which consists of the top tank is shown in Fig. 1. It is reminded that in configuration 1, the pump feeds into tank 1 and that tank 2 is not considered at all. Therefore, the input to the process is the voltage to the pump and its output is the water level in tank 1. The mathematics model of the single tank system is determined by relating the volumetric inflow rate $f_{i1}$ into tank to the outflow rate $f_{o1}$ leaving through the hole at the tank bottom.

The volumetric inflow rate and the outflow rate to tank 1 can be expressed as [23]

$$f_{i1}=K_pV_p \tag{1}$$

$$f_{o1}=A_{o1}v_{o1} \tag{2}$$

Where $A_{o1}$ is the outlet cross sectional are and $v_{o1}$ is the tank 1 outflow velocity. $K_p$ is the pump volumetric flow constant and $V_p$ is the actual pump voltage. The outflow velocity by using Bernoulli's equation is

$$v_{o1}=\sqrt{2}\sqrt{gL_1} \tag{3}$$

where g is the gravitational constant on earth. As a remark, the cross-section area of tank 1 outlet hole can be calculated by

$$A_{o1}=\frac{1}{4}\pi D_{o1}^2 \tag{4}$$

In Eq. (4) $D_{o1}$ is the tank 1 outlet diameter. Using Eq. (3) the outflow rate from tank 1 given in Eq. (3) becomes,

$$f_{o1}=A_{o1}\sqrt{2}\sqrt{gL_1} \tag{5}$$

Moreover, using the mass balance principle for tank 1, we obtain the following first-order differential equation in $L_1$:

$$A_{t1}\left(\frac{\partial}{\partial t}L_1\right)=f_{i1}-f_{o1} \tag{6}$$

Where, $A_{t1}$ is tank 1 inside cross-section area. Substituting Eqs. (1) and (2) into Eq. (6) and it can be rearranged in the following form for the tank 1 system

$$\frac{\partial}{\partial t}L_1=\frac{K_pV_p-A_{o1}\sqrt{2}\sqrt{gL_1}}{A_{t1}} \tag{7}$$

### 2. Coupled Tank Model (Configuration 2)

A schematic of the coupled tank plant is depicted in Fig. 2. In configuration 2, the pump feeds into tank 1, which in turn feeds into tank 2. As far as tank 1 is concerned, the same equation as the ones previously developed in section (2.1) is applied. However, the water level equation of motion in tank 2 still needs to be derived. In the coupled tanks, the system states are the level $L_1$ in tank 1 and the level $L_2$ in tank 2. The outflow rate from tank 2 can be expressed as

$$f_{o2}=A_{o2}v_{o2} \tag{8}$$

Tank 2 outflow velocity by using Bernoulli's equation is

$$v_{o2}=\sqrt{2}\sqrt{gL_2} \tag{9}$$

As a remark, the cross-section area of tank 2 outlet hole can be calculated by

$$A_{o2}=\frac{1}{4}\pi D_{o2}^2 \tag{10}$$

Using Eqs. (9) and (10) the outflow rate from tank 2 given in Eq. (8) becomes

$$f_{o2}=A_{o2}\sqrt{2}\sqrt{gL_2} \tag{11}$$

Using Eq. (5) the inflow rate to tank 2 is as follows:

$$f_{i2}=A_{o1}\sqrt{2}\sqrt{gL_1} \tag{12}$$

Moreover using the mass balance principle for tank 2, we obtain the following first-order differential equation in $L_2$:

$$A_{t2}\left(\frac{\partial}{\partial t}L_2\right)=f_{i2}-f_{o2} \tag{13}$$

Substituting Eqs. (12) and (11) into Eq. (13) and it can be rearranged in the following form for the tank 2 system:

$$\frac{\partial}{\partial t}L_2=\frac{-A_{o2}\sqrt{2}\sqrt{gL_2}+A_{o1}\sqrt{2}\sqrt{gL_1}}{A_{t2}} \tag{14}$$

## SLIDING MODE CONTROL (SMC) OF A COUPLED TANK SYSTEM

A controller can be designed for a system represented in the regular form as [21]

$$\dot{x}_1 = f_1(x_1, x_2, t) \tag{15}$$

$$\dot{x}_2 = f_2(x_1, x_2, t) + B_2(x_1, x_2, t)u(t) \tag{16}$$

In this state space description, $x_1 \in \Re^{n-m}$, $x_2 \in \Re^m$ and **B** is an m×**m** nonsingular matrix. The goal is to drive states of the system in the set *S* defined by:

$$S=\{x: \tau(t) - \xi(x) = \varepsilon(x, t) = 0\} \tag{17}$$

where $\tau(t)$ is the time dependent part of the sliding function, containing reference inputs to be applied to the controller. $\xi(x)$ denotes the state dependent part of the sliding function, $\varepsilon(x, t)$. The derivation of the control involves the selection of a Lyapunov function $V(\varepsilon)$ and a desired form of derivative of the Lyapunov function such that closed-loop system is stable. The selected Lyapunov function is [20-22]

$$V = \frac{1}{2}\varepsilon^T \varepsilon \tag{18}$$

which is positive definite, and its derivative is

$$\dot{V} = -\varepsilon^T \dot{\varepsilon} \tag{19}$$

The solution $\varepsilon(x, t)=0$ will be stable if time derivative of the Lyapunov function can be expressed as [21]

$$\dot{V} = -\varepsilon^T D \varepsilon \tag{20}$$

where D is a positive definite matrix. Thus, the derivative of the Lyapunov function will be negative definite and this will ensure the stability. Eqs. (19) and (20) lead to

$$\varepsilon^T (D\varepsilon + \dot{\varepsilon}) = 0. \tag{21}$$

A solution for this equation is

$$D\varepsilon + \dot{\varepsilon} = 0. \tag{22}$$

The expression for derivative of the sliding function is

$$\frac{d\varepsilon}{dt} = \frac{d\tau}{dt} - \frac{d\xi}{dt} \tag{23}$$

Where

$$\xi = G_1 x_1 + G_2 x_2 \tag{24}$$

$G_1 \in \Re^{n \times n-m}$ and $G_2 \in \Re^{n \times m}$ are gain matrices, and

$$\frac{d\xi}{dt} = G_1 \frac{\partial x_1}{\partial t} + G_2 \frac{\partial x_1}{\partial t} \tag{25}$$

First, equivalent control is found by $\dot{\varepsilon}=0$ and using Eq. (23) as

$$\dot{\varepsilon} = \dot{\tau} - \dot{\xi} = \dot{\tau} - (G_1 f_1 + G_2 f_2 + G_2 B_2 u_{eq}) = 0 \tag{26}$$

$$u_{eq} = (G_2 B_2)^{-1}(\dot{\tau} - G_2 f_2 - G_1 f_1) \tag{27}$$

Second, using Eq. (24) the control input to the system can be found by the following:

$$\dot{\varepsilon} = -D\varepsilon = \dot{\tau} - \dot{\xi} \tag{28}$$

$$\dot{\tau} - (G_2 f_2 + G_1 f_1 + G_2 B_2 u) = -D\varepsilon \tag{29}$$

And the result of the short algebra can be written as

$$u = u_{eq} + (G_2 B_2)^{-1} D\varepsilon \tag{30}$$

Third, from time derivative of the sliding function

$$\frac{d\varepsilon}{dt} = \dot{\tau} - (G_2 f_2 + G_1 f_1 + G_2 B_2 u) \tag{31}$$

Multiplying both sides with $(G_2 B_2)^{-1}$

$$(G_2 B_2)^{-1}\frac{d\varepsilon}{dt} = (G_2 B_2)^{-1}(\dot{\tau} - G_2 f_2 - G_1 f_1) - u \tag{32}$$

And by using (29)

$$(G_2 B_2)^{-1}\frac{d\varepsilon}{dt} = u_{eq} - u \tag{33}$$

Finally, when this equation is substituted in (30) the control is found as

$$u(t) = u(t^-) + (G_2 B_2)^{-1}\left(\frac{d\varepsilon}{dt} + D\varepsilon\right)$$

$$t = t^- + \Delta, \Delta \to 0 \tag{34}$$

The value of the control at the instant t is calculated from the value at the time $(t-\Delta)$ and the weighed sum of the control error $\varepsilon$ and its time derivative. Control (34) is continuous function everywhere except in the points of discontinuity of the function $\varepsilon(x, t)$. When these equations are adapted for level control system shown in Fig. 3, the following equation can be written for the outer loop as

$$L_{1ref}(t) = L_{1ref}(t^-) + (G_2 B_2)^{-1}\left(\frac{d\varepsilon_2}{dt} + D\varepsilon_2\right) \tag{35}$$

The control equation can be adapted for the inner loop as

$$V_p(t) = V_p(t^-) + (G_1 B_1)^{-1}\left(\frac{d\varepsilon_1}{dt} + D\varepsilon_1\right) \tag{36}$$

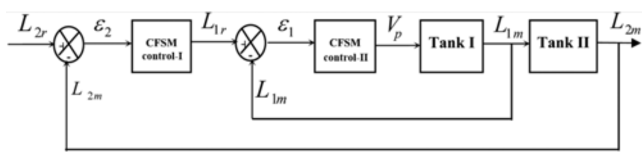


**Fig. 3. Schematic diagram of the level control system.**

**Fig. 4. The coupled tank experimental setup.**

where

$$\varepsilon_2 = L_{2r} - L_{2m},\ \varepsilon_1 = L_{1r} - L_{1m} \tag{37}$$

$B_1$ and $B_2$ are multiplier coefficients of control signals of the system model. A generalized control structure of coupled tank system is depicted in Fig. 3. As seen from the figure, there are two controllers of which the first is used to determine reference level of tank 1 and second one is used to determine pump voltage command to ensure the tank 1 water level to feeds tank 2.

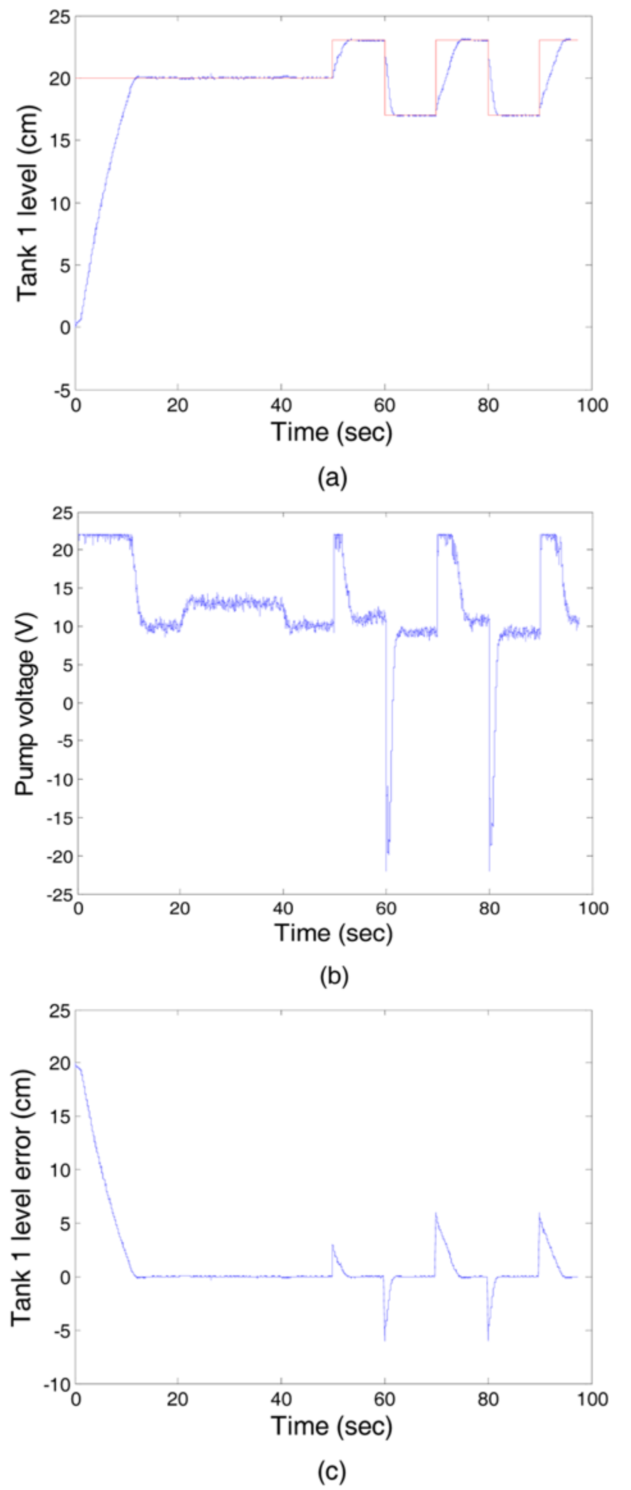


**Fig. 5. Experimental results for step+square reference in configuration 1.**
(a) Actual and reference level of tank 1, (b) Pump voltage command ($V_p$), (c) Level error

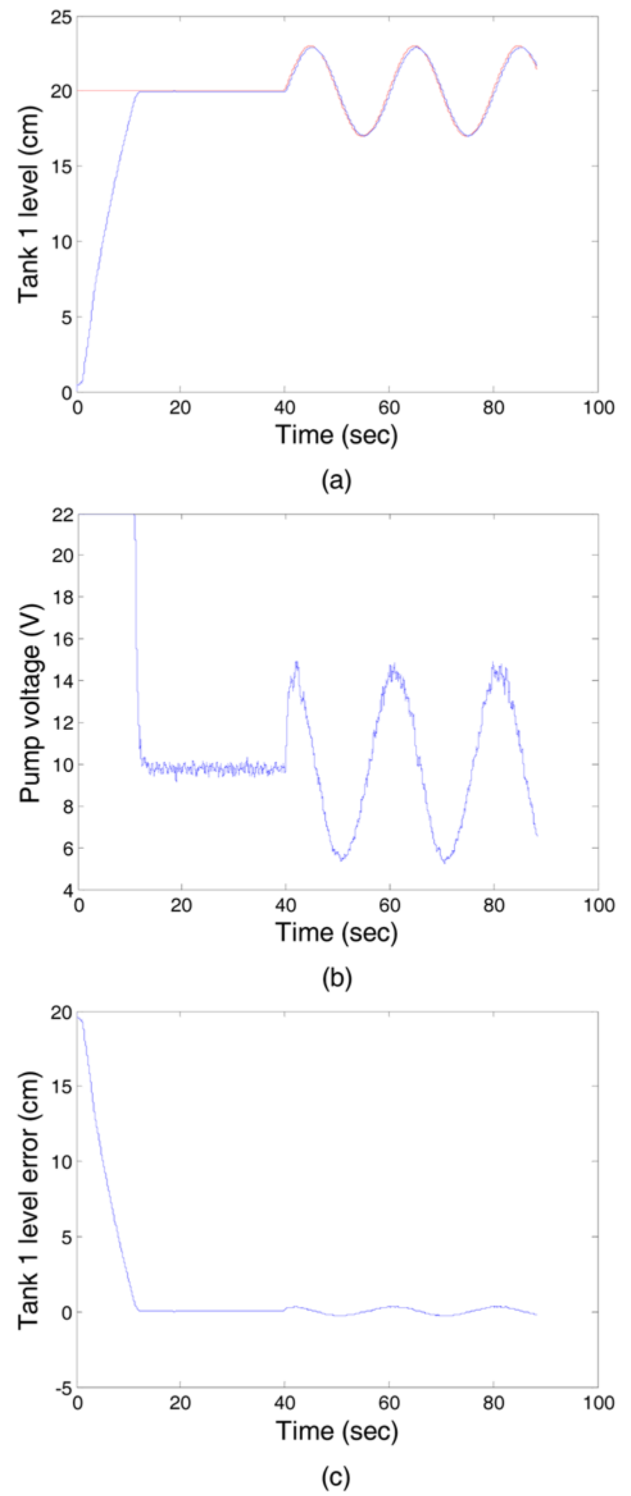


**Fig. 6. Experimental results for step+sinusoidal reference in configuration 1.**
(a) Actual and reference level of tank 1, (b) Pump voltage command ($V_p$), (c) Level error

## EXPERIMENTAL RESULTS

The experimental setup of the coupled-tank plant is depicted in Fig. 4. The coupled-tank specialty module is a “Two-Tank” module consisting of a pump with a water basin and two tanks of uniform cross sections. Such an apparatus forms an autonomous closed and recirculating system. The two tanks, mounted on the front plate, are configured such that flow from the first (top) tank can flow into second (bottom) tank. Flow from the second tank flows into the main water reservoir. In each one of the two tanks, liquid is with-

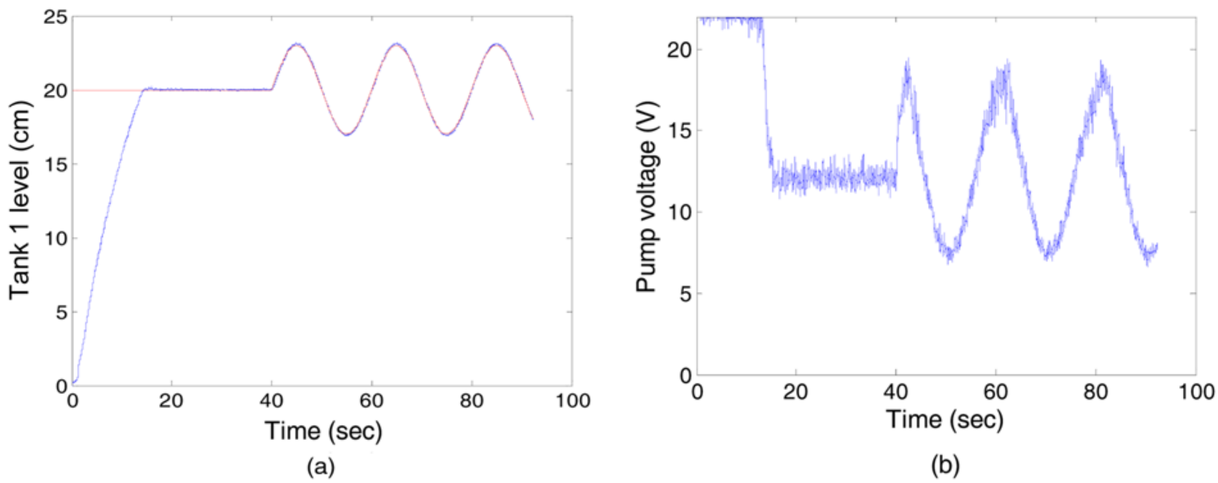


**Fig. 7. Experimental results for step sinusoidal reference under parameter variations in configuration 1.**

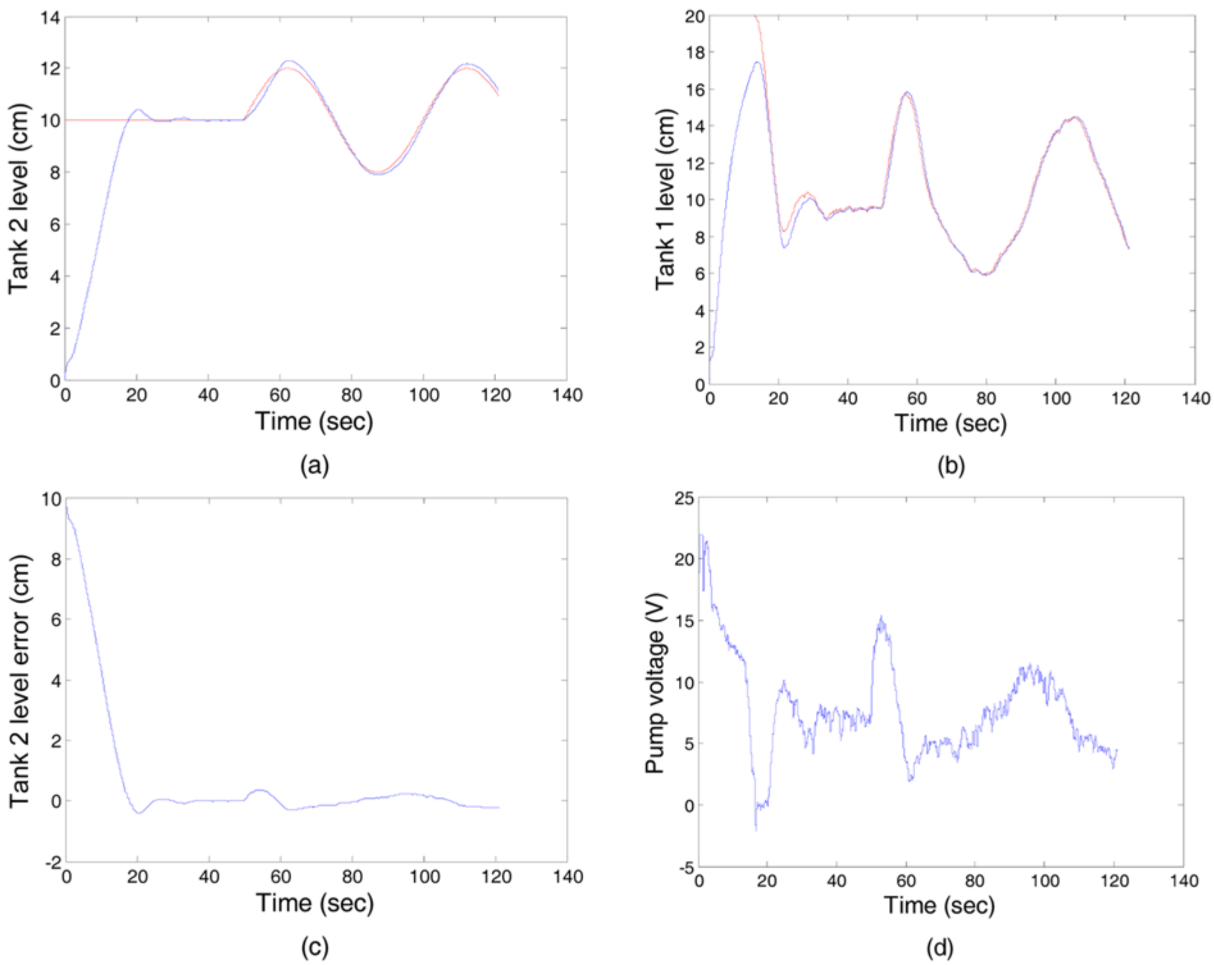


**Fig. 8. Experimental results for step+sinusoidal reference in configuration 2.**
(a) Actual and reference levels of tank 2, (b) Actual and reference levels of tank 1, (c) Level error for tank 2, (d) Pump voltage command ($V_p$)

drawn from the bottom through an outflow orifice. The outlet pressure is atmospheric. To introduce a disturbance flow, the first tank is also equipped with a drain tap so that, when opened, flow can be released directly into the water basin. The water level in each tank is measured using a pressure-sensitive sensor located at the bottom of the each tanks.

Experimental results are presented in this section to show the performance of the controller for both single and coupled tank system. We have applied a chattering-free sliding mode controller to water level control system, and the experimental results are shown in Figs. 5-7. In the first experiment, the performance of the controller is tested for step+square and step+sinusoidal level references for single tank configuration. The square wave reference is important because the control system is tested for a sudden increasing and decreasing liquid level in a period. For the cases of step+square level reference shown in Fig. 5, a big disturbance is applied between 20 to 40 seconds to show the proposed controller responses against the undesired circumstances. As seen from Fig. 5(a) and 5(b), the proposed controller increases the pump voltage to overcome the disturbance quickly and follows the reference level perfectly. On the other hand step+ sinusoidal level reference, shown in Fig. 6, is applied to determine the controller response for a reference changing continuously. The liquid level of the tank 1 could keep up with the desired ones quickly and the actual level follows the reference trajectory with small error. As shown in Fig. 5(b) and 6(b) the actual pump voltage command is continuous and overcomes reference variations and disturbances.

The performance of CFSMC under parameter variations of the system is shown in Fig. 7. The diameter of tank 1's outer is replaced with a bigger one (by 20%) and application of Fig. 6(b) is repeated without changing control signal parameters and related result is shown in Fig. 7. As seen from Fig. 7(b), the magnitude of the control signal increases to overcome the parameter changes. The results show the robustness of CFSMC under parameter variations of the system.

In the second experiment, a step sinusoidal reference level is chosen for tank 2 to show the responses of the coupled tanks conditions. Proposed controllers produce reference level of tank 1 and voltage command of pump to guarantee that tank 2 actual liquid level track the given reference. It can be observed on Fig. 8(a) and 8(b) that actual levels of tank 1 and tank 2 follow references, and the pump voltage command does not include discontinuous signal of general SMC that causes chattering. The experimental results show that the proposed control scheme is capable of improving the tracking precision effectively and the responses of the liquid level control system are very good and satisfactory.

## CONCLUSION

A chattering-free sliding mode controller is applied for the liquid level control of a coupled tank system. Level control algorithm is implemented to single and coupled tanks conditions with complex references, respectively. The experimental results strongly show that the proposed approach provides good position tracking performance with high precision as well as good robustness against disturbance and changes of references. The resulting control signals are continuous and smooth compared to discontinuous control signals of general sliding mode method. The control signal eliminates chattering and proves the viability of the control method.

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