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RAPID COMMUNICATION

Fractional order integrator for the relay feedback identification of a process Nyquist point in the third quadrant

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Abstract−A fractional order integrator can be used for the relay feedback identification of a process Nyquist point in the third quadrant, and to implement the fractional order integrator, it is often approximated by integer order systems. Here, instead of the usual rational transfer function approximation of the fractional order integrator in the relay feedback system, a simple analytic method which utilizes the on-off characteristics of relay output is proposed. Simulation results show that the proposed method can find process Nyquist points in the third quadrant without worrying about the approximation errors and ranges of the fraction order integrator.

Key words: Fractional Order Integrator, Relay Feedback Identification, Nyquist Point in the Third Quadrant

INTRODUCTION

Recent proportional-integral-derivative (PID) controllers have autotuning features and, for them, relay feedback identifications of Astrom and Hagglund [1] are often used [5]. The conventional relay feedback identification method finds the process ultimate information, a Nyquist point at the phase of −180°. Lee et al. [8] proposed a method to decrease errors in the conventional relay feedback identification with some characteristic areas of relay feedback responses. Parameters of PID controllers can be determined with the process ultimate information through the Ziegler-Nichols like tuning rules. However, the frequency corresponding to the phase of −180° is too large to tune for some PID controllers, especially for PI controllers widely used in the field [4,6,12].

For the identification of process frequency response in the third Nyquist quadrant by the relay feedback method, dynamic elements such as time delay and low pass filter can be used. Their phase shifts are dependent on the oscillation frequency, which is not known in advance, and they are adjusted adaptively to obtain the frequency response at the given phase shift. For a non-adaptive method, Friman and Waller [4] proposed a two-channel relay feedback method for the Nyquist point at the given phase shift.

Jiri et al. [6] proposed a method to use the fractional order integrator for the frequency response at a given phase shift. Fractional order systems have recently been studied in various areas [2,9,10]. To implement fractional order systems, they are approximated by rational transfer functions with integer orders [3]. However, Tavazoei and Haeri [11] show that rational approximations should be used cautiously because dynamical behaviors of some approximated models can be completely different from those of the original fractional order systems. Here, a simple analytical method to implement a fractional order integrator is proposed for the relay feedback identification of the process frequency response at a given phase shift.

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RELAY WITH A FRACTIONAL-ORDER FILTER FOR A NYQUIST POINT IN THE THIRD QUADRANT

Consider the relay feedback system of Fig. 1 for the identification of Nyquist point of the process G(s) in the third quadrant. Here a fractional order integrator

$$F_m(s)=\frac{1}{s^m} \tag{1}$$

is used to obtain a given phase lag. The fractional order integrator has a phase lag of

$$\angle F_m(j\omega)=-\frac{m\pi}{2} \tag{2}$$

Hence, by choosing an appropriate m, a Nyquist point in the third quadrant can be obtained [6]. For implementation of the fractional order integrator of Eq. (1), it is often approximated by a finite dimensional system with rational transfer function [3,6]. Here, utilizing the on-off characteristics of the relay output, a simple analytic method to implement the fractional order integrator is proposed.

The Laplace transform of process input u(t) is

$$U(s)=F_m(s)U_r(s)=\frac{1}{s^m}U_r(s) \tag{3}$$

where $U_r(s)$ and U(s) are Laplace transforms of the relay output and the fractional order integrator output, respectively. Let the relay output be switched on at $t_0(=0)$, off at t_1, on at t_2, and so on, and h be the amplitude of relay output. Applying the convolution theorem about the multiplication of Laplace transforms [7], we have

$$\begin{aligned}u(t)&=\int_0^t f_m(t-\tau)u_r(\tau)d\tau\\&=h\int_{t_0}^{t_1} f_m(t-\tau)d\tau-h\int_{t_1}^{t_2} f_m(t-\tau)d\tau+\cdots+(-1)^n h\int_{t_n}^{t} f_m(t-\tau)d\tau\\&=h(q_m(t-t_0)-2q_m(t-t_1)+2q_m(t-t_2)+\cdots+(-1)^n 2q_m(t-t_n))\end{aligned} \tag{4}$$

where

$$f_m(t)=\frac{t^{m-1}}{\Gamma(m)}:\ \text{inverse of } F_m(s)$$

$$q_m(t)=\int_0^t f_m(t)dt=\frac{t^m}{m\Gamma(m)} \tag{5}$$

Because 4-5 relay oscillations are used for the identification, the number of terms needed for u(t) is small and its computation will be simple.

For example, when m=0.5, we have $f_m(t)=\frac{1}{\sqrt{\pi t}}$, $q_m(t)=\int_0^t \frac{1}{\sqrt{\pi t}}dt=\frac{2\sqrt{t}}{\sqrt{\pi}}$ [7] and

$$u(t)=\frac{2h}{\sqrt{\pi}}(\sqrt{t-t_0}-2\sqrt{t-t_1}+2\sqrt{t-t_2}+\cdots+(-1)^n 2\sqrt{t-t_n}) \tag{6}$$

and the phase shift of the fractional order integrator with m=0.5 is -45^o. Fig. 1 shows typical responses of the relay output and the fractional order integrator. Here the first relay output is on and off for a fast start of oscillation.

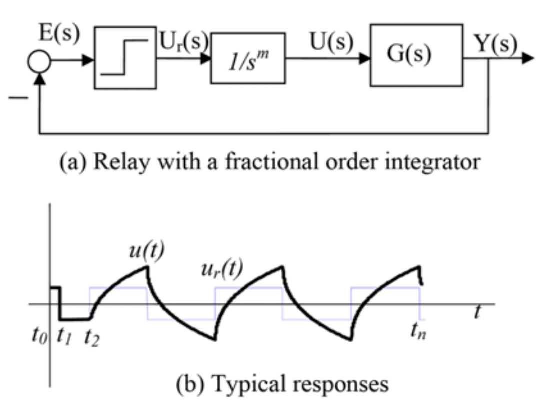

Fig. 1. Relay feedback system with a fractional order integrator.

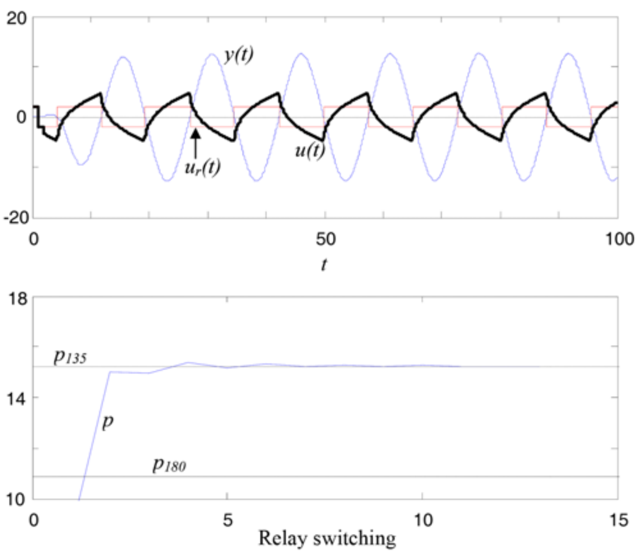

Fig. 2. Responses of the relay feedback system with a fractional order integrator ($G(s)=5/(s+1)^6$, m=0.5). Here p_{135} and p_{180} are the oscillation periods corresponding to the process phase angles at -135^o and -180^o, respectively.

The relay feedback system of Fig. 1 will oscillate approximately at the frequency ω of $\angle F_m(j\omega)G(j\omega)=-\pi$. Hence the oscillation period p is such that

$$\angle G(j\omega)=-\pi-\angle F_m(j\omega)=-(1-m/2)\pi,\ \omega=2\pi/p \tag{7}$$

Since $|F_m(j\omega)G(j\omega)|=\pi a/(4h)$ where a is the amplitude of process output [1], the amplitude ratio of $G(j\omega)$ at the oscillation frequency is

$$|G(j\omega)|=\frac{\pi a\omega^m}{4h} \tag{8}$$

Fig. 2 shows responses for the process $G(s)=5/(s+1)^6$ with the fractional order integrator of m=0.5. We can see that the oscillation period is near 15.169 such that $\angle G(j\omega)=-3\pi/4$. For the identification of this Nyquist point, Jiri et al. used a 4-th order filter

$$F_m(s)=\frac{0.000007285s^4+0.006125s^3+0.4134s^2+3.402s+3.108}{0.0003158s^4+0.06627s^3+1.544s^2+4.385s+1} \tag{9}$$

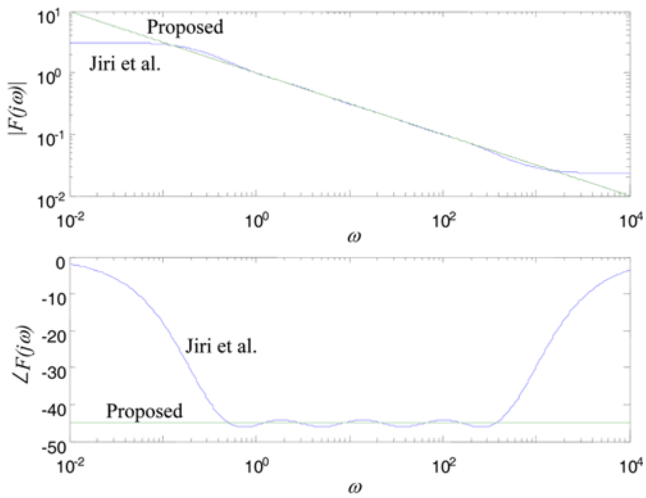

Fig. 3. Bode plots of filters for the Nyquist point at -135^o.

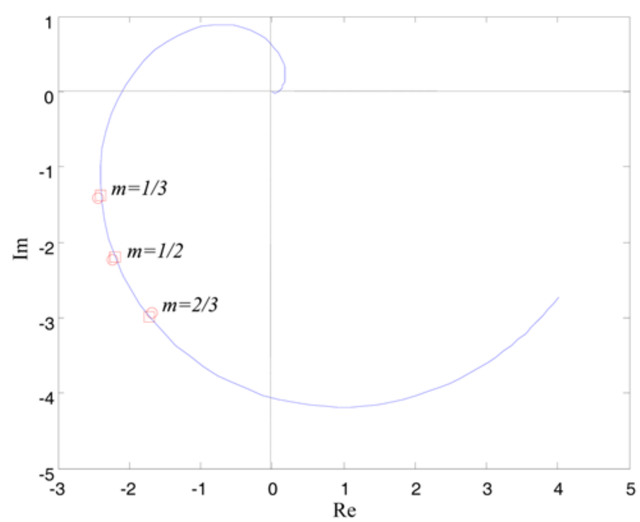

Fig. 4. Nyquist points identified ($G(s)=5/(s+1)^6$, sampling time for simulations: 0.01, circle: identified, square: exact).

approximating the fractional order filter $F_{0.5}(s)=1/s^{0.5}$. Fig. 3 shows Bode plots of this approximate filter and the proposed fractional order filter. The filter by Jiri et al. [6] shows phase errors below 1.1° for frequencies between 0.4 and 450. These phase errors of the filter do not cause any significant estimation errors. However, the oscillation frequency should be inside the valid frequency range, and because the oscillation frequency of process is not known in advance, time scaling may be required for some applications. The proposed method does not require such time scaling and the computation is much simpler.

Fig. 4 shows some Nyquist points identified with various filter order m. PID controllers can be tuned with these Nyquist points and tuning rules in Friman and Waller [4].

CONCLUSION

A fractional order integrator introduced in the relay feedback system can be used to obtain a process frequency response at a given phase shift. Utilizing the on-off characteristics of relay, an analytic method to implement a fractional order integrator in the relay feedback system is proposed. It is not needed to worry about bad effects due to approximated rational models usually used for the implementation of fractional order systems.

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