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Process identification method using relay feedback and backward integrals

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Abstract−We propose a new process identification method that combines the two methods of the relay feedback to activate the process and the backward integrals to estimate the model parameters. Novel deviation variables are introduced to incorporate the case that the initial part of the process is unsteady-state without sacrificing the dynamic information included in the initial part, while the previous approaches assign zero-weighting to the initial parts, resulting in loss of the dynamic information included in the initial part. The final cyclic-steady-state part of the process input and output data is chosen as the reference of the deviation variables. The proposed method can estimate the model parameters analytically by using the backward integrals and the least squares method.

Key words: Relay Feedback, Backward Integral, Process Identification, Parameter Estimation, Dynamic Information, Initial-unsteady-state

INTRODUCTION

The process activation and the model parameter estimation techniques are core parts of process identification. The aim of process activation research is to activate important dynamic information of the process with perturbing the process minimally. The purpose of the parameter estimation technique is to obtain the most accurate model parameters from the activated process data. Both process activation and model parameter estimation should be considered simultaneously for the best process identification.

One of the most efficient process activation methods is the relay feedback method. It automatically determines the finishing time by checking the number of the cycles of the relay on/off, resulting in a minimal process activation time. Usually, the process activation can be stopped after 3 or 4 cycles. Also, the relay feedback method can intensively activate dynamic information around specified ranges of the phase angle, and it shows good robustness to the process uncertainty such as measurement noises and disturbances [1-11]. For these reasons, the relay feedback method is chosen to activate the process in this research and an efficient parameter estimation method will be designed for the relay feedback signals.

The model parameter estimation methods can be divided into two categories. One is to obtain the discrete-time model in the form of the difference equation, and the other is to obtain the continuous-time model in the form of the differential equation. In this research, we will focus on the continuous-time model identification method because it can incorporate small, large and irregular sampling time in a systematic way [12,13]. Until now, many continuous-time model identification methods have been proposed. One of them is to obtain the continuous-time parametric model using the model reduction technique after obtaining the frequency response data [14]. The describing function analysis [1], Fourier analysis and several different types of Fourier transform [15-17] have been proposed to calculate the frequency responses of the process. Once the frequency responses are estimated by the above-mentioned methods, it is straightforward to estimate the model parameters of the continuous-time differential equation model using the model reduction method [14].

A variety of the continuous-time model identification methods that directly obtain the differential equation model have also been proposed, such as Walsh functions [18], block pulse functions [19], Legendre polynomials [20], multiple integrators [21,22] and prediction error method [12]. But, they are only applicable to the cases that the initial part of the process data is zero-steady-state. To overcome the initial-unsteady-state problem, linear integral transforms, macro-difference expressions, ball-shaped weighting integral transforms and polynomial integral transforms have been developed [13, 14,23-25]. These methods assign zero weighting to the initial part to solve the initial-unsteady-state problem. This means that the previous methods sacrifice important dynamic information included in the initial part. It would be remarkable if a novel approach were successfully developed to solve the initial-unsteady-state problem without sacrificing the initial dynamic information.

In this paper, we choose the relay feedback method to activate the process and the backward integrals to estimate the model parameters. The proposed method defines new deviation variables of which the references are the final part of the activated process data to solve the initial-unsteady-state problem. Then, the model parameters can be estimated by the least squares method after applying the backward integrals to the general linear differential equation model. The proposed process identification method can incorporate the initial-unsteady-state problem in an efficient way without sacrificing the dynamic information included in the initial part of the process data, while the previous approaches assign zero-weighting to the initial part, resulting in loss of the dynamic information included in the initial parts.

PROPOSED PROCESS IDENTIFICATION METHOD

Many previous process identification methods are only applicable to the cases that the initial part of the process input-output data

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is zero-steady-state. So, several identification methods assigning zero-weighting to the initial part of the activated process input-output data have been proposed to overcome the problems. These strategies result in a new problem of sacrificing the important dynamic information included in the initial part. The proposed method uses a novel deviation variable to solve the initial-unsteady-state problem without sacrificing the initial dynamic information and the backward integrals to estimate the model parameters of the continuous-time differential equation model using the least squares method without solving any nonlinear optimization problems in a repetitive manner.

1. Process Activation and New Deviation Variables

In this research, it is assumed that the process input (u(t)) and output (y(t)) are activated by a biased-relay feedback method as shown in Fig. 1 or 2. Here, the reference values for the relay on-offs are changed around t=7.0 from 0.5 to −0.5. The final parts of Figs. 1 and 2 are both cyclic-steady-state. But, the initial part of Fig. 1 is different from that of Fig. 2. The initial part of Fig. 1 is steady-state while that of Fig. 2 is unsteady-state. Previous process identification methods have limitations in estimating the model parameters from the input-output measurements of Fig. 2 because its initial part is unsteady-state as mentioned in the introduction section. New deviation variables will be defined in this research to overcome the initial unsteady-state problem as follows:

First, obtain the input-output data (u(t), y(t)) as shown in Fig. 1 or 2 using a biased-relay. Second, generate the new signals (denoted by $y_r(t)$, $u_r(t)$) of Fig. 3 by repeating the last cycle of the final cyclic-steady-state part of Fig. 1 or 2. Third, obtain the deviation variables (denoted by $y_{sub}(t)$, $u_{sub}(t)$) of Fig. 4 by subtracting the signals ($y_r(t)$, $u_r(t)$) of Fig. 3 from those (y(t), u(t)) of Fig. 1. That is, $y_{sub}(t)=y(t)-y_r(t)$ and $u_{sub}(t)=u(t)-u_r(t)$. Finally, obtain the final deviation variables (denoted by $y_{final}(\bar{t})$, $u_{final}(\bar{t})$) of Fig. 5 by reversing the time axis of Fig. 4. That is, $\bar{t}=t_f-t$. Here, t_f is the final time of the process input-output data of Fig. 4. Note that the initial part of the deviation variables of Fig. 5 becomes zero-steady-state. Now, it is possible to obtain the model parameters from the final deviation variables ($y_{final}(\bar{t})$,

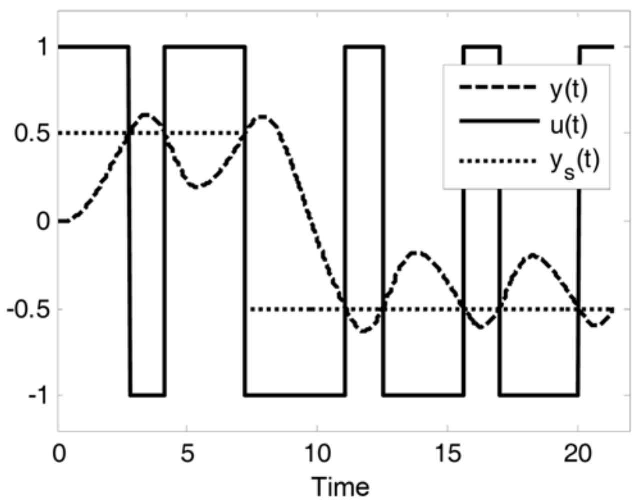

Fig. 1. The responses of the relay feedback method for the initial-steady-state process.

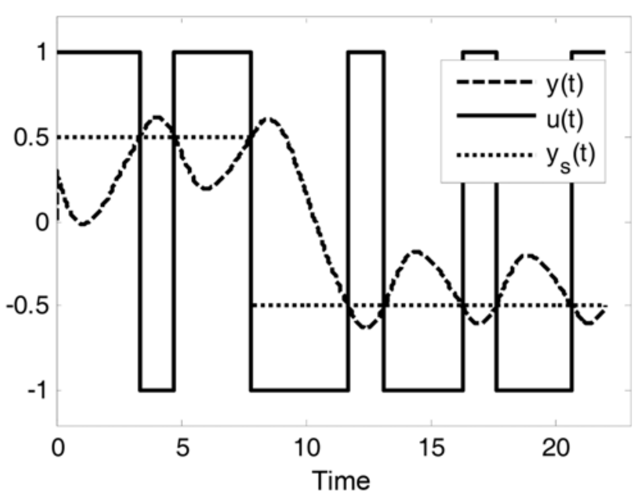

Fig. 2. The responses of the relay feedback method for the initial-unsteady-state process.

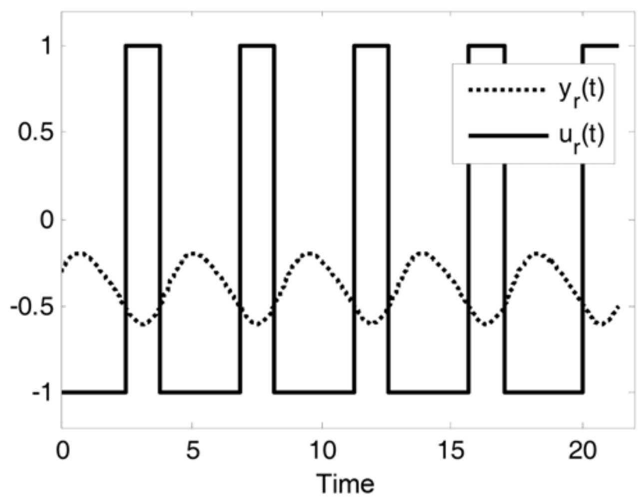

Fig. 3. The signals generated by repeating the last cycle in the signals of Fig. 1.

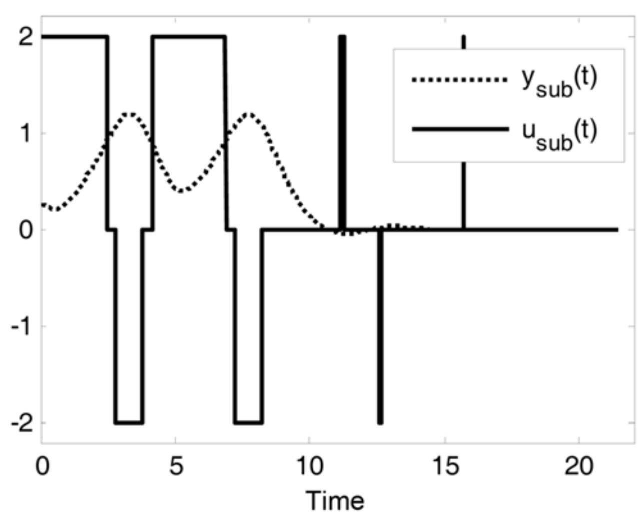

Fig. 4. The signals obtained by subtracting Fig. 3 from Fig. 1.

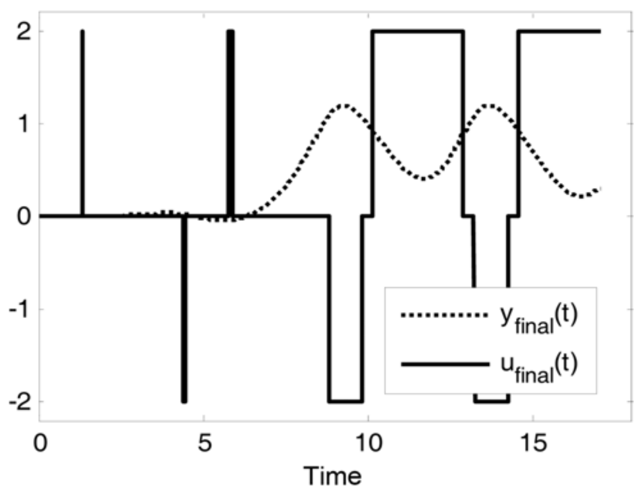

Fig. 5. The new signals obtained by reversing the time of Fig. 4.

$u_{final}(\bar{t})$) without the initial-unsteady-state problem as shown in the next section.

2. Proposed Model Parameter Estimation Method

Consider the following general linear time-invariant process.

$$G_p(s)=\frac{b_m s^m+b_{m-1}s^{m-1}+\cdots+b_1 s+b_0}{a_n s^n+a_{n-1}s^{n-1}+\cdots+a_1 s+1} \quad (1)$$

Eq. (1) can be rewritten to the following differential equation [14].

$$a_n\frac{d^n y(t)}{dt^n}+a_{n-1}\frac{d^{n-1}y(t)}{dt^{n-1}}+\cdots+a_1\frac{dy(t)}{dt}+y(t)$$
$$=b_m\frac{d^m u(t)}{dt^m}+b_{m-1}\frac{d^{m-1}u(t)}{dt^{m-1}}+\cdots+b_0 u(t)+B_0 \quad (2)$$

Where, y(t) and u(t) are the process output and input, respectively. B_0 is a bias term to incorporate static components. The purpose of the proposed parameter estimation method is to calculate the model parameters (a_n, a_{n-1}, $\cdots$, a_1, b_m, b_{m-1}, $\cdots$, b_0, B_0) from the process data activated by the biased-relay. Because $y_r(t)$ and $u_r(t)$ in Fig. 3 are the signals obtained by repeating the last cycle of the final cyclic-steady-state part of Fig. 2, the signals are the solutions of the differential equation of Eq. (2). So, the following differential equation is valid.

$$a_n\frac{d^n y_r(t)}{dt^n}+a_{n-1}\frac{d^{n-1}y_r(t)}{dt^{n-1}}+\cdots+a_1\frac{dy_r(t)}{dt}+y_r(t)$$
$$=b_m\frac{d^m u_r(t)}{dt^m}+b_{m-1}\frac{d^{m-1}u_r(t)}{dt^{m-1}}+\cdots+b_0 u_r(t)+B_0 \quad (3)$$

Eq. (4) can be obtained by subtracting Eq. (3) from Eq. (2).

$$a_n\frac{d^n y_{sub}(t)}{dt^n}+a_{n-1}\frac{d^{n-1}y_{sub}(t)}{dt^{n-1}}+\cdots+a_1\frac{dy_{sub}(t)}{dt}+y_{sub}(t)$$
$$=b_m\frac{d^m u_{sub}(t)}{dt^m}+b_{m-1}\frac{d^{m-1}u_{sub}(t)}{dt^{m-1}}+\cdots+b_0 u_{sub}(t)+B \quad (4)$$

Where, $y_{sub}(t)=y(t)-y_r(t)$ and $u_{sub}(t)=u(t)-u_r(t)$. The bias term of B is added to Eq. (4) to incorporate uncertainties such as disturbances and errors introduced in generating the signals of Fig. 3. Eq. (4) corresponds to the governing differential equation of Fig. 4. By reversing the time axis (that is, $\bar{t}=t_f-t$) of Fig. 4, the new input-output data of Fig. 5 can be obtained. And, the following differential equation of Eq. (5), applicable to the input-output data of Fig. 5 can be derived straightforwardly from Eq. (4) with considering that two the time directions are apposite (that is, $\bar{t}=t_f-t$). Here, $\bar{t}$ and t are times of Fig. 4 and Fig. 5, respectively, and t_f is the final time of Fig. 4. Note that the initial part of the signals of Fig. 5 is zero-steady-state. So, the initial-unsteady-state problem is just solved.

$$a_n(-1)^n\frac{d^n y_{final}(\bar{t})}{d\bar{t}^n}+a_{n-1}(-1)^{n-1}\frac{d^{n-1}y_{final}(\bar{t})}{d\bar{t}^{n-1}}+\cdots+y_{final}(\bar{t})$$
$$=b_m(-1)^m\frac{d^m u_{final}(\bar{t})}{d\bar{t}^m}+b_{m-1}(-1)^{m-1}\frac{d^{m-1}u_{final}(\bar{t})}{d\bar{t}^{m-1}}$$
$$+\cdots+b_0 u_{final}(\bar{t})+B \quad (5)$$

Let us define the backward integrals on $y_{final}(\bar{t})$ and $u_{final}(\bar{t})$ in Fig. 5 as follows:

$$I_y^k(\bar{t})=(-1)^{n-k}\underbrace{\int_0^{\bar{t}}\iint\cdots\int}_{k} y_{final}(\alpha_1)\,\underbrace{d\alpha_1\cdots d\alpha_k}_{k},\ k=1,2,\cdots,n$$

$$I_u^k(\bar{t})=(-1)^{n-k}\underbrace{\int_0^{\bar{t}}\iint\cdots\int}_{k} u_{final}(\alpha_1)\,\underbrace{d\alpha_1\cdots d\alpha_k}_{k},\ k=1,2,\cdots,m \quad (6)$$

Then, Eq. (7) is obtained by integrating Eq. (5) n-times.

$$a_n(-1)^n y_{final}(\bar{t})+a_{n-1}I_y^1(\bar{t})+\cdots+a_1 I_y^{n-1}(\bar{t})+I_y^n(\bar{t})$$
$$=b_m I_u^{n-m}(\bar{t})+b_{m-1}I_u^{n-m+1}(\bar{t})+\cdots+b_1 I_u^{n-1}(\bar{t})+b_0 I_u^n(\bar{t})+B\bar{t}^n/n! \quad (7)$$

The backward integrals of Eq. (6) can be calculated with usual numerical integration methods.

On the other hand, one frequency response data of the process can be estimated from the cyclic-steady-state part of Fig. 1 or 2 by the following Fourier analysis [14,15].

$$G_p(i\omega_{css})=\frac{\int_{css}^{t_{css}+P_{css}}\exp(-i\omega_{css})y(t)dt}{\int_{css}^{t_{css}+P_{css}}\exp(-i\omega_{css})u(t)dt} \quad (8)$$

Where, ω_{css} and P_{css} are the frequency and the period of the final cyclic-steady-state in Fig. 1 or 2, respectively. $G_p(i\omega_{css})$ is the frequency response of the process at the cyclic-steady-state frequency. Then, Eq. (9) and Eq. (10) are obtained from Eq. (1) by inserting the available frequency response data of Eq. (8) into Eq. (1) as follows:

$$G_p(i\omega_{css})=\frac{b_m(i\omega_{css})^m+b_{m-1}(i\omega_{css})^{m-1}+\cdots+b_1(i\omega_{css})+b_0}{a_n(i\omega_{css})^n+a_{n-1}(i\omega_{css})^{n-1}+\cdots+a_1(i\omega_{css})+1} \quad (9)$$

$$a_n=\frac{-a_{n-1}(i\omega_{css})^{n-1}-\ldots-a_1(i\omega_{css})-1}{(i\omega_{css})^n}$$
$$+\frac{b_m(i\omega_{css})^m+\cdots+b_1(i\omega_{css})+b_0}{G_p(i\omega_{css})(i\omega_{css})^n} \quad (10)$$

Then, the optimization problem of Eq. (11) for the proposed process identification method can be easily derived from Eq. (7) and Eq. (10). Now, the model parameters of a_{n-1}, $\cdots$, a_1, b_m, b_{m-1}, $\cdots$, b_0, B can be estimated by solving the optimization problem of Eq. (11) by the least squares method and a_n is obtained by Eq. (10).

$$\begin{aligned}\underset{a,b,B}{\mathbf{MIN}}\sum_{k=0}^{k=n_k}\Bigg\| & a_{n-1}\left(I_y^1(\bar{t}_k)-\frac{(-1)^n y_{final}(\bar{t}_k)}{(i\omega_{css})}\right)+\cdots+a_1\left(I_y^{n-1}(\bar{t}_k)-\frac{(-1)^n y_{final}(\bar{t}_k)}{(i\omega_{css})^{n-1}}\right)\\ & +I_y^n(\bar{t}_k)-\frac{(-1)^n y_{final}(\bar{t}_k)}{(i\omega_{css})^n}-b_m\left(I_u^{n-m}(\bar{t}_k)-\frac{(-1)^n y_{final}(\bar{t}_k)}{G_p(i\omega_{css})(i\omega_{css})^{n-m}}\right)\\ & -\cdots-b_1\left(I_u^{n-1}(\bar{t}_k)-\frac{(-1)^n y_{final}(\bar{t}_k)}{G_p(i\omega_{css})(i\omega_{css})^{n-1}}\right)\\ & -b_0\left(I_u^n(\bar{t}_k)-\frac{(-1)^n y_{final}(\bar{t}_k)}{G_p(i\omega_{css})(i\omega_{css})^n}\right)-B\bar{t}_k^n/n!\Bigg\|,\quad \bar{t}_k=k(\bar{t}_f/n_k) \end{aligned}\tag{11}$$

Where, $\bar{t}_f$ and n_k+1 are the final time of Fig. 5 and the number of the data points for the least squares method, respectively. ||•|| denotes 2-norm.

2-1. Remarks

1. The previous approaches sacrifice the dynamics included in the initial part of the activated process input and output data to incorporate the case of the initial-unsteady-state. Meanwhile, the proposed method solves the initial-unsteady-state problem without sacrificing the initial part by introducing new deviation variables and backward integrals.

2. The proposed method estimates the model parameters in an analytic way without solving any nonlinear optimization problems in a repetitive manner.

3. The effects of unmeasurable static disturbances on the parameter estimation are completely removed in the proposed method since the new deviation variables completely delete the static disturbances and the Fourier analysis also removes the effects of the static disturbance on estimating the frequency response data of the cyclic-steady-state [14,15].

4. Because the proposed method uses the integrals, good robustness to measurement noises can be secured for usual cases that the variance of the noise does not change much with respect to the sampling time [12].

5. The proposed method can be applied to unstable or integrating processes in the same way as done in [26]. For example, if we choose the transfer function of the process in the form of $G_p(s)=(b_m s^m+\cdots+b_1 s+b_0)/(a_n s^n+\cdots+a_1 s-1)$ and modified all the equations from Eq. (2) to Eq. (11) according to the simple sign change (from +1 to −1) of the transfer function, unstable processes can be easily identified by the proposed approach.

SIMULATION STUDY

Several examples are simulated to demonstrate the performance of the proposed method.

1. Example 1

Consider the following third-order plus time delay process. Fig. 1 shows the process input-output data activated by a biased-relay. In this example, the initial part is zero-steady-state as shown in Fig. 1.

$$G_p(s)=\frac{\exp(-0.1s)}{(s+1)^3}\tag{12}$$

Fig. 6 shows the performance of the proposed process identification method. As expected, the proposed method shows almost perfect modeling performances.

2. Example 2

The same process of Example 1 is activated as shown in Fig. 2.

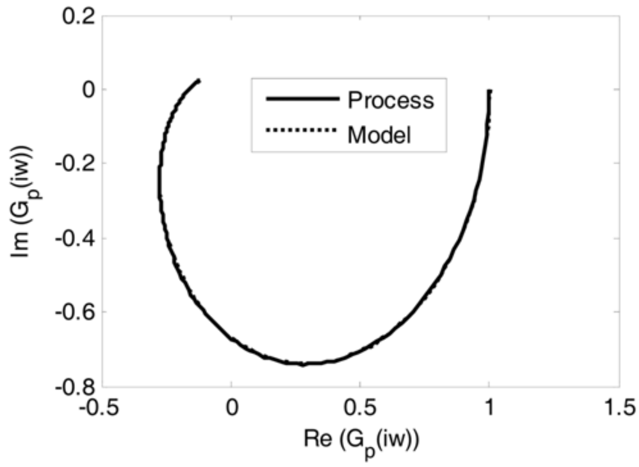

Fig. 6. Identification results of the proposed method for the initial-steady-state process.

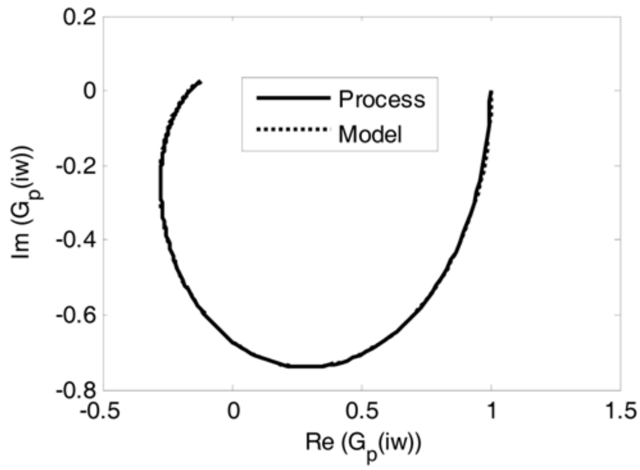

Fig. 7. Identification results of the proposed method for the initial-unsteady-state process.

The initial values of the process are as follows:

$$\left.\frac{d^2y(t)}{dt^2}\right|_{t=0}=0.1,\ \left.\frac{dy(t)}{dt}\right|_{t=0}=-0.5,\ y(0)=0.3,$$
$$\left.\frac{du(t)}{dt}\right|_{t=-0.1}=0.0,\ u(-0.1)=1.0\tag{10}$$

This means that the initial part is unsteady-state as shown in Fig. 2, different from the initial zero-steady-state part of Fig. 1. Fig. 7 shows the performance of the proposed process identification method. As expected, it can effectively incorporate the case of the initial-unsteady-state.

3. Example 3

Consider the process of Example 2 with uniformly distributed measurement noises between −0.1 and 0.1 as shown in Fig. 8. Fig. 9 demonstrates that the proposed process identification method shows acceptable robustness to the measurement noises.

4. Example 4

Consider the process of Example 1 with the static input disturbance of 0.1. Fig. 10 shows the activated process input and output.

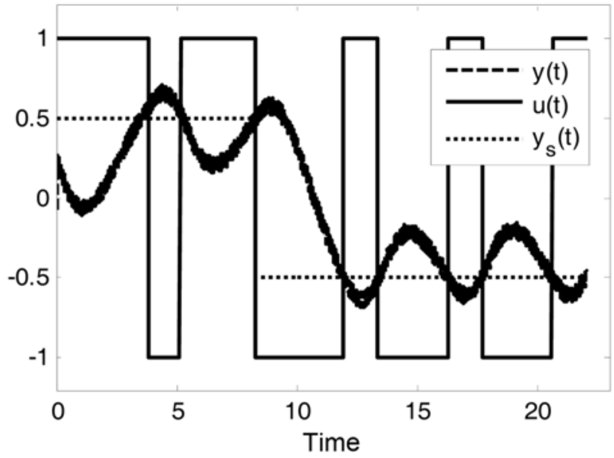

Fig. 8. The responses of the relay feedback method for measurement noises.

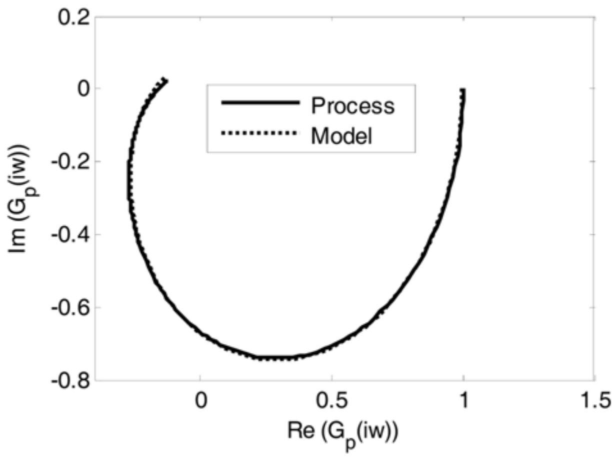

Fig. 9. Identification results of the proposed method for measurement noises.

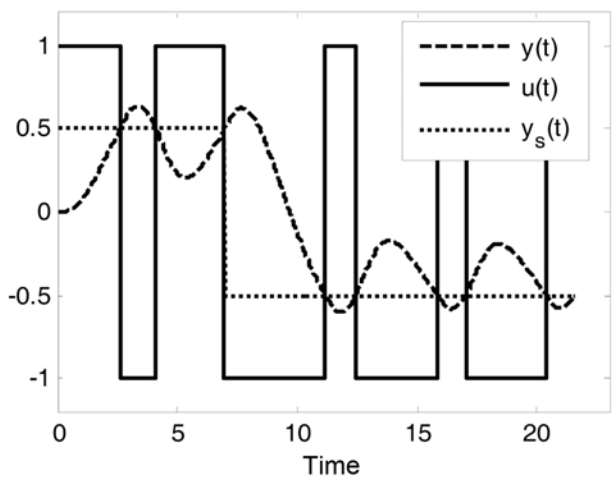

Fig. 10. The responses of the relay feedback method for the static disturbance.

Fig. 11 demonstrates that the proposed process identification method can completely reject the effects of the static disturbance as expected.

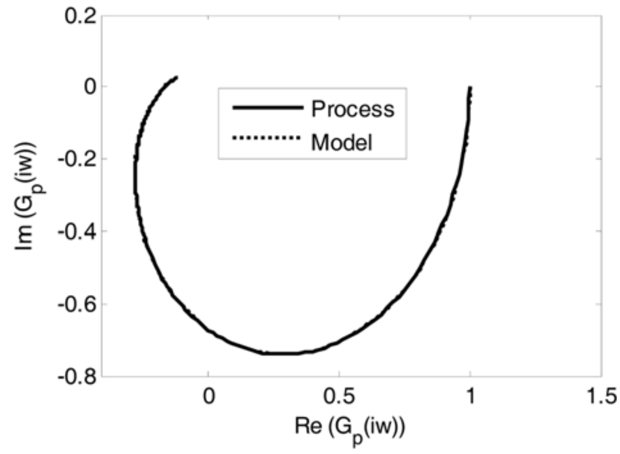

Fig. 11. Identification results of the proposed method for the static disturbance.

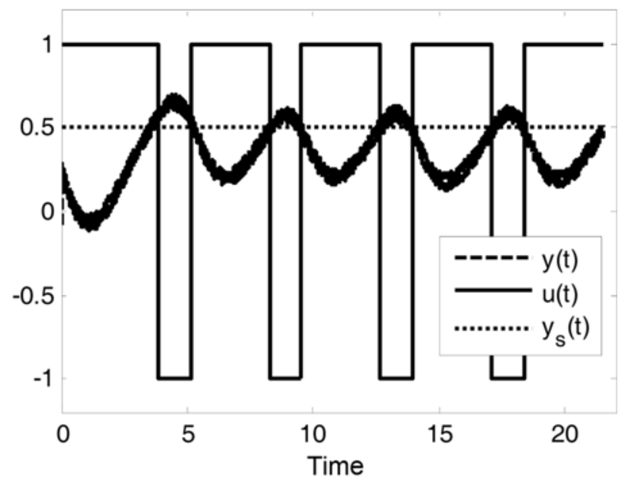

Fig. 12. The responses of the relay feedback method for comparison between the proposed method and the previous method.

5. Example 5

Consider the process input-output data activated by a biased-relay feedback method as shown in Fig. 12. In this case, roughly speaking, many frequency components are included in the initial part, while the cyclic-steady-state part includes only two frequency components corresponding to zero and fundamental frequency. So, the previous methods sacrificing the dynamic information included in the initial part cannot provide acceptable model accuracy. Fig. 13 compares the performances of the proposed process identification method and the previous method [14,25] that sacrifices the initial dynamic information to incorporate the case of the initial-unsteady-state. As expected, the proposed method shows much better performance because it solves the initial-unsteady-state problem without sacrificing the dynamic information included in the initial parts of the process data.

CONCLUSIONS

This research proposes a new process identification method using two the methods of the relay feedback to activate the process and

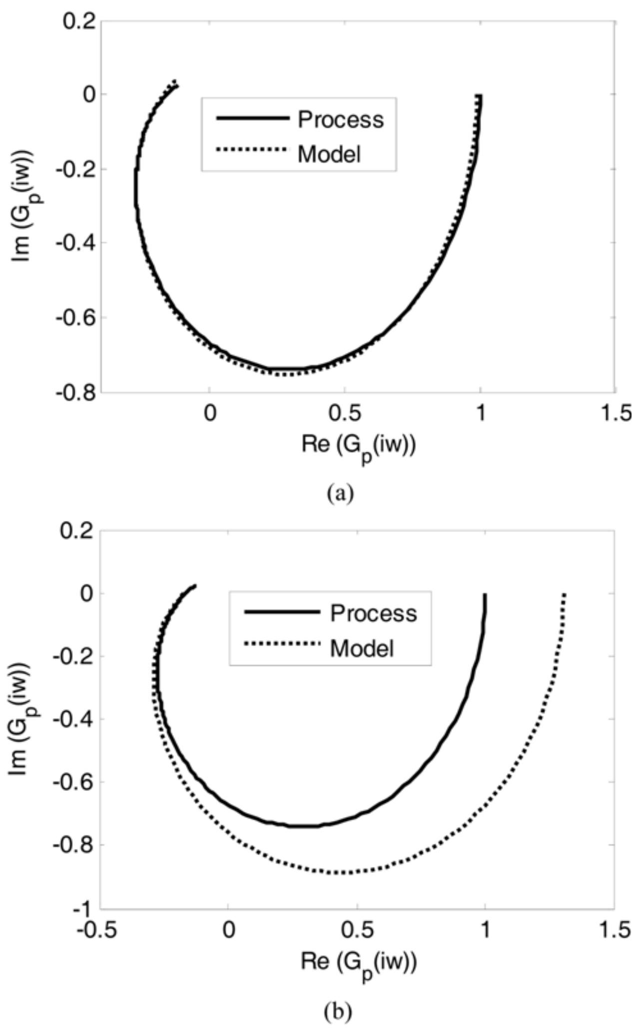

Fig. 13. Identification results of the proposed method (a) and the previous method (b).

the backward integrals to estimate the model parameters. It can solve the initial-unsteady-state problem without sacrificing the dynamic information included in the initial parts of the process data by introducing novel deviation variables, of which the references are the final cycle of the process input and output data. The proposed method can also estimate the model parameters in an analytic way without solving any nonlinear optimization problem in a repetitive manner by using the least squares method and the backward integrals. Simulation results successfully demonstrate the acceptable accuracy and robustness of the proposed approach.

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NOMENCLATURE

a_i, b_i, B, B_0 : model parameters
$G_p(s)$: transfer function
i : unitary pure imaginary number
$I_y^k(\bar{t})$, $I_u^k(\bar{t})$: backward integrals
n, m : order of transfer function
n_k+1 : the number of data points
P_{css} : period of cyclic-steady-state part
$\bar{t}$, t_f, t : time
u(t) : process input
$u_r(t)$: reference of deviation variables
$u_{sub}(t)$, $u_{final}(\bar{t})$: deviation variables
y(t) : process output
$y_r(t)$: reference of deviation variables
$y_{sub}(t)$, $y_{final}(\bar{t})$: deviation variables

Greek Letters

ω_{css} : frequency of cyclic-steady-state part

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