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A scenario-based robust framework for short-term and long-term operation planning problems

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Abstract−This paper discusses a scenario-based scheme to address process operation problems under variations. As an illustration, two typically encountered problems, a short-term operation planning problem and a long term supply chain planning problem, are investigated considering the effect of variations. Their mathematical programming models explicitly reflecting the variations in the form of multiple scenarios with their probabilities are presented to provide robust decisions in the face of variations. It is hoped that the proposed framework can be a reminder of the importance of the scenario-based approach in the current uncertain business circumstances.

Key words: Scenarios, Robust, Variation, Scheduling, SCM (Supply Chain Management)

INTRODUCTION

The presence of variation cannot be excluded in process operation decision-making. Industry practices are attracted by the deterministic approach since it provides the ideally best result that peoples hope to see. However, variations do occur in reality and the hoped-for best result is in fact hard to realize. Instead of focusing on computing the deterministic best case that is not likely to actually happen, it would be more realistic and practically important to compute the best robust operation plan that is insensitive to actual variations and try to put them into execution.

The major work in the PSE community has been focused on computing the deterministic values of the process operation problem. How to select the best among the many discrete alternatives is a challenge, but another issue not to be missed is to compute decisions that are insensitive to the variation. Process operation problems may be transposed into the following optimization problem:

$$\begin{aligned}&\min f(x, y, e) \\ &\text{s.t.} \\ &\quad g(x, y, e)\le 0 \\ &\quad h(x, y, e)=0\end{aligned} \tag{1}$$

where x denotes continuous variables and y binary variables and e represent parameters. While the values of some parameters may be known and fixed, some are often varying and known only after they are realized in the future. Therefore, (1) can be of the following form:

$$\begin{aligned}&\min f(x, y, e^F, \theta) \\ &\text{s.t.} \\ &\quad g(x, y, e^F, \theta)\le 0 \\ &\quad h(x, y, e^F, \theta)=0\end{aligned} \tag{2}$$

where e^F denote fixed parameters and θ, varying parameters. Although we cannot know the exact value of θ, their potential set of candidate values, called scenarios, and their probabilities may be available. (2) can thus be reformulated into the following deterministic decision-making model:

$$\begin{aligned}&\min \sum_s p_s f_F(x, y, e^F, \theta_s) \\ &\text{s.t.} \\ &\quad g(x, y, e^F, \theta_s)\le 0 \quad s=1, \ldots, S \\ &\quad h(x, y, e^F, \theta_s)=0 \quad s=1, \ldots, S\end{aligned} \tag{3}$$

where s denotes scenarios and the sum of p_s is 1.

Based upon this scenario-based framework, two typically encountered process systems engineering operation problems will be addressed in this paper. One is a short-term operation planning problem and the other is a long-term macro-scale supply chain network planning problem. This paper aims to convey the message that operation problems should be addressed by considering the presence of variation regardless of their scale such as long-term or short-term decision-making problems. As an explicit tool to address the variation in process operation problems, a number of methods such as stochastic programming, parametric programming, scenario-based method, etc. are already available (see the recent review by [1]). This paper employs a scenario-based method because it is a simple but straightforward methodology to reflect the variation in exploring the complicated process operation decision-making problems. The scenario-based approach may not be a state-of-the-art computation methodology. But it is thought that its application still provides valuable insights on process operation. This paper aims to emphasize the importance of a scenario-based framework in the current uncertain business circumstances.

For pharmaceutical and high value-added complicated chemical process industries that are subject to variations of various types, the first task would be to estimate the effect of the variation on the problem at hand. To respond to such situations, their effects on the performance should be evaluated. It is hoped that insight obtained by using the proposed scenario-based framework will increase our understanding on the variations in the process industries.

In the literature, many researchers have studied robustness ([2-7] and so on). Pinedo and Weiss [2], De et al. [3] and Vin and Ierapetritou [6] suggested a robust scheduling scheme considering their ex-

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pected value and variance of uncertain parameters of which distributions are known. Pinedo and Weiss [2] discussed heuristic rules such as longest expected processing time first (LEPF), shortest expected processing time first (SEPF) policy, etc. They pointed that in case of same or similar mean processing time the two previous policies could not give the best solution. They emphasized the usage of largest variance first (LVF) policy by calculating variance of processing times. De et al. [3] also considered the expectation and variance of uncertain processing time. They made a partial sequence and used an algorithm to estimate the whole variance. However, the addressed two papers did not calculate the variance of objective functions but evaluated those by each distribution of uncertain parameter.

Bok et al. [8] employed the concept of robustness in the planning problem. They considered the presence of variation as multiple scenarios and they minimized the standard deviation of the mean of the objective function. Vin and Ierapetritou [6] solved scheduling problems with multiperiod uncertain demand by STN (state task network) based formulation. The objective function was the expected makespan under uniformly distributed demands. They tested standard deviation as a robust metric while its average was iteratively taken by means of makespan and deterministic makespan, i.e., the most possible makespan. They considered not single period but multiperiod as the number of possible scenarios with a view to making it robust. Standard deviation was used not as optimizing objective either but as robustness metric. Petkov and Maranas [4] suggested an equivalent deterministic formulation method of a stochastic problem by Monte Carlo Sampling of uncertain parameters.

In formulating stochastic problems and solving the corresponding problems involving uncertain parameters where their values are given as a set of parameters, the corresponding result is obtained in a 'wait and see' solution. Most previous studies handling scenario-based stochastic problems took the expected value as the objective function of the problem. However, the expected value itself denotes the sum of objective values multiplied by individual probabilities of actual occurrence. It implies nothing but currently calculated the biggest value. It does not work when an unexpected event actually happens. Therefore, an additional criterion must be considered. In quality engineering, there is 'Taguchi' technology that is a skill fitting a value representing the quality of a certain product to a given target value. This technique also includes smaller-the-better and larger-the-better characteristics. Del Castillo and Montgomery [9] and Vining and Myer [10] laid the foundation for RSM (response surface method) using Taguchi technology. Lin and Tu [11] proposed dual RSM, which optimizes the target value simultaneously considering the variance of the value preventing from the oscillation by extra causes, which implies distribution of objective values near a stable value. Instead of the former intention that the objective value was to stick to the target value regardless of oscillation, they emphasized stability and robustness while admitting small deviation from the target.

The motivation from their work is shown in Fig. 1, where the estimated mean response curve is denoted by ω_μ and the estimated standard deviation response curve is denoted by ω_σ. The purpose is to find an optimal set of conditions such that ω_μ will be close to the target value T, while the standard deviation ω_σ is kept small. Suppose the target for the mean is T as indicated. In this case, Vining and Myer [10]'s approach first restricts ω_μ to T. Four points (A, B, C and D) satisfy this restriction. Among them, point A has the minimum variance and thus is the "optimal" setting. Studying the behavior of ω_μ and ω_σ, it is easy to see that point E is a better choice than point A. By introducing a little bias, we may reduce the variance a great deal. In fact, point E minimizes the mean square error (MSE= $(\omega_\mu - T)^2 + \hat{\omega}_\sigma^2$).

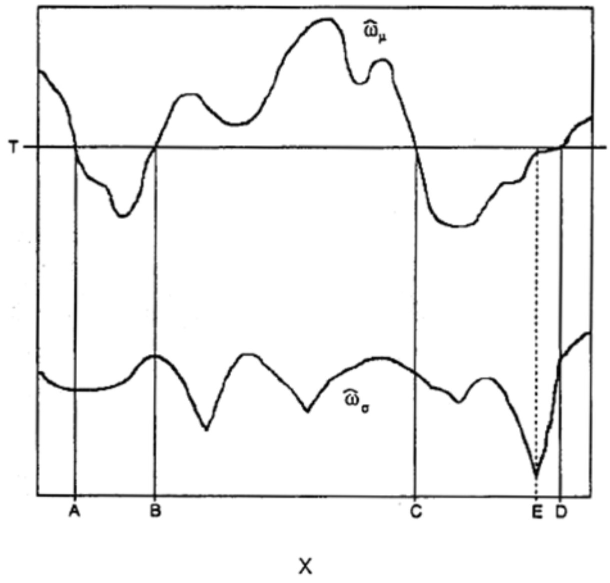


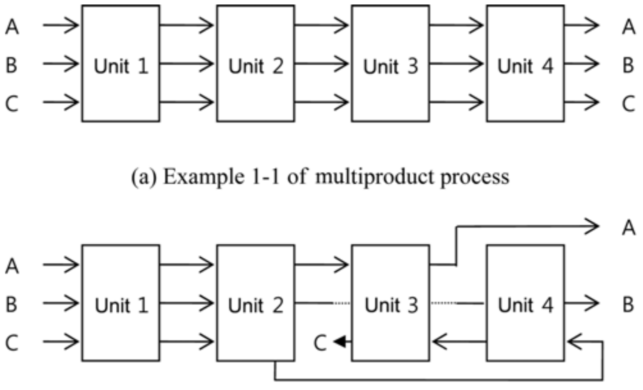
Fig. 1. Schematic Diagram illustrating Importance of Variation (source: [11]).

In the context of scheduling, robustness can be defined as a measure of resilience of the scheduling objective to change like parameter variations and disruptive events. Schedules are computed satisfying a variety of different objectives such as makespan minimization or maximization of profit or production. The most important determinant variable of scheduling problems is sequence. Briefly, robust scheduling of a stochastic problem is taken as to find a sequence that minimizes the influences by any events.

The rest of this paper is constructed as follows: First, the short-term operation scheduling decision problems are considered under the processing time variations. The long-term macro scale planning problem is then considered, considering the raw material price variations. Numerical examples are presented to illustrate the applicability of the proposed framework with some remarks.

SCENARIO-BASED ROBUST SHORT-TERM OPERATION PLANNING APPROACH

Many researchers have been interested in scheduling in the context of providing deterministic formulations consuming fewer constraints and less resource or providing better result [12-15,45]. Some are interested in integrating hierarchically divided decision-making frameworks in supply chains such as Munawar et al. [16], Puigjaner et al. [40], Bonfill et al. [17]. At the same time, some works have been proposed using other types of computational methodologies such as Till et al. [18], Majozi and Zhu [19], Ghaeli et al. [20]. There are also a significant number of studies that address the scheduling problem with other process operation problems like planning such as Moreno et al. [21], Moreno and Montagna [22]. Dogan and



(a) Example 1-1 of multiproduct process

(b) Example 1-2 of multipurpose process

Fig. 2. Schematic diagram of example.

Table 1. Distribution and probability of processing times

K	J	$\mathbf{PT}_{kjs}/\mathbf{P}_s$			
A	1	15/0.4	16/0.1	17/0.5	-
	2	8/0.55	9/0.25	11/0.2	-
	3	11	-	-	-
	4	18	-	-	-
B	1	14/0.05	16/0.6	18/0.05	19/0.3
	2	8/0.45	10/0.35	13/0.05	15/0.15
	3	12	-	-	-
	4	9	-	-	-
C	1	15/0.45	17/0.3	20/0.25	-
	2	7/0.4	8/0.6	-	-
	3	17	-	-	-
	4	7	-	-	-

Grossmann [23].

In addressing problems involving uncertain parameters, we may not know their exact values, but their potential values with the probabilities are more likely to be known, for example, from reference or the previous experiment data.

Two cases of batch operations are discussed to test the robustness of schedule. Consider a multiproduct batch process dealing with three products A, B and C through four process units as graphically described in Fig. 2(a) as noted as 1-1.

It is known that processing times in equipment units 1 and 2 are not deterministic due to the characteristics of products and process. The processing times are summarized in Table 1. The number of the scenarios is 846 (3×3×4×4×3×2).

In the rest of this paper, it is necessary to distinguish between 'objective value' and 'objective function' for clarification purpose. 'Objective value' represents the value of interest, which is makespan for scheduling problems. 'Objective function' denotes the set of objective values to be minimized or maximized in the optimization model like expected makespan plus variance. In this paper, in order to develop a framework that provides robust operation policy in the context of schedule and plan, a scenario-based modeling formulation is constructed. Specifically, in effort to provide such robust result, different types of objective functions have been proposed in this paper. Eqs. (4), (5a) and (5b) are the illustrations of such objective functions. The corresponding mathematical programming formulation is as follows.

Objective function:

$$\sum_s P_s \cdot MS_s \tag{4}$$

$$\sum_s P_s \cdot MS_s + \left(\sum_s P_s \cdot MS_s^2 - \sum_s (P_s \cdot MS_s)^2\right) \tag{5a}$$

$$\sum_s P_s \cdot MS_s + \sqrt{\sum_s P_s \cdot MS_s^2 - \sum_s (P_s \cdot MS_s)^2} \tag{5b}$$

where P_s implies probability and MS_s is makespan under scenario s, respectively. Then, Eq. (4) is the expected makespan based on all the scenarios. Eq. (5a) denotes the sum of the expected value and its variance and (5b), the sum of the expected value and its standard deviation, respectively.

Subject to:

$$\sum_i \mathbf{w}_{ki} = \mathbf{1} \quad \forall \mathbf{k} \tag{6}$$

$$\sum_k \mathbf{w}_{ki} = \mathbf{1} \quad \forall \mathbf{k} \tag{7}$$

where w_{ki} is binary variable, which is valued 1 when product k is produced in i th sequence, otherwise 0. Product k must be processed one of all sequences, which Eq. (6) implies. Eq. (7) denotes that one of the products must be processed in each sequence.

In unit j at ith sequence, the completion time of ith product, C_{ijs} must be greater than the completion time of directly previous product, $C_{i-1,j,s}$ plus its processing time, PT_{kjs} for each scenario s.

$$\mathbf{C}_{ijs} \geq \mathbf{C}_{i-1,j,s} + \sum_k \mathbf{PT}_{kjs} \cdot \mathbf{w}_{ki} \quad \forall \mathbf{i}, \mathbf{j}, \mathbf{s} \tag{8}$$

Each *i*th product is processed only after the task of the previous unit is completed.

$$\mathbf{C}_{ijs} \geq \mathbf{C}_{i,j-1,s} + \sum_k \mathbf{PT}_{kjs} \cdot \mathbf{w}_{ki} \quad \forall \mathbf{i}, \mathbf{j} \geq \mathbf{2}, \mathbf{s} \tag{9}$$

The makespan is the completion time of the last product produced.

$$MS_s \geq C_{ijs} \quad \forall i, j \tag{10}$$

As another example, consider a multipurpose batch process where each product is produced by a different sequence of units (for deterministic model formulation, refer to the work by [24]). As illustrated in Fig. 2(b), product A is completed at unit 3. Product B skips unit 3 and production of C is done at unit 3 after unit 4. Binary variable X_{kij} is 1 when product k is produced *i*th task of unit j, otherwise 0. The first index of completion time C is not sequence i but product k. The objective function is the same as the multiproduct problem.

Objective function: (1), or (2a), or (2b)
Subject to: (7) and

$$\sum_{i(j)} \mathbf{w}_{kij} = \mathbf{1} \quad \forall \mathbf{k}, \mathbf{j}(\mathbf{k}) \tag{11}$$

Each product k must be processed in its tasking unit j.

$$\sum_k \mathbf{w}_{k,i,j} = \mathbf{1} \quad \forall \mathbf{i}(\mathbf{j}), \mathbf{j} \tag{12}$$

A certain product must be processed in each step i of unit j.

$$\mathbf{C}_{k,j,s} \geq \mathbf{PT}_{k,j,s} \quad \forall \mathbf{k}, \mathbf{j}(\mathbf{k}), \mathbf{s} \tag{13}$$

$$C_{k,j,s} - RT_{k,j,s} \ge C_{k,j',s} \quad \forall k,\ j(k),\ j'(k,j) \tag{14}$$

If unit j treats product k, it can start after the task of previous unit j′ is completed.

$$C_{k,j,s} - PT_{k,j,s} \ge C_{k',j,s} - M\cdot(2 - w_{k,i,j} - w_{k',i-1,j}) \quad \forall k,\ k'(\neq k),\ i(j)(\geq 2),\ j \tag{15}$$

In unit j, product k is processed after the previous product k′ is completed. In Eq. (15), M denotes a large parameter. Only in case that product k′ is processed in i-1th and product k is processed ith task of unit j, constraint (15) becomes valid; otherwise it is invalid due to the effect of a sufficiently large positive number M.

The proposed model is employed to compute the solutions of the above two cases, Example 1-1 and 1-2. Fig. 3 shows the result of Example 1-1 and Fig. 4 for Example 1-2, respectively. The different values of makespans are obtained depending on the different objective function. As can be seen in Fig. 3, the makespan is scattered when we consider all scenarios in minimizing its expected value. When the objective function includes other factors such as variance and standard deviation of the makespan in addition to expected cost term, the result shows that the shape of the distribution is changed.

There are a number of issues worthy of the further comment. First, we assumed that we could have robust solutions of optimization

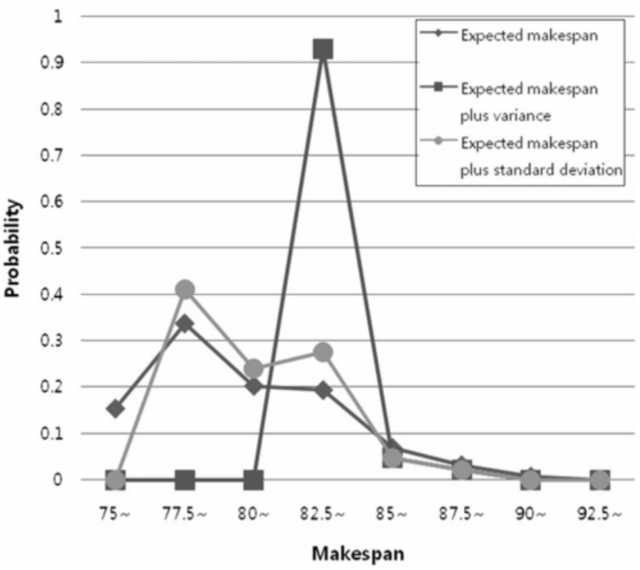


Fig. 3. Result of Example 1-1: Distributions of makespan with different objective function (unit: Hr).

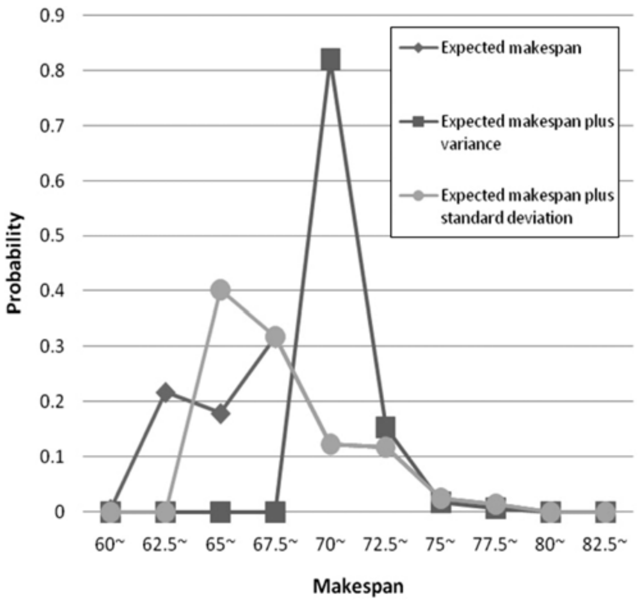
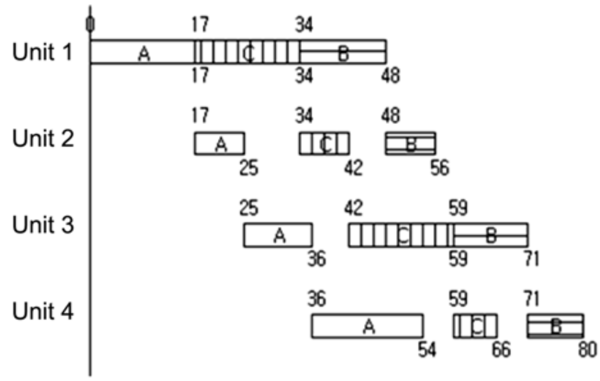
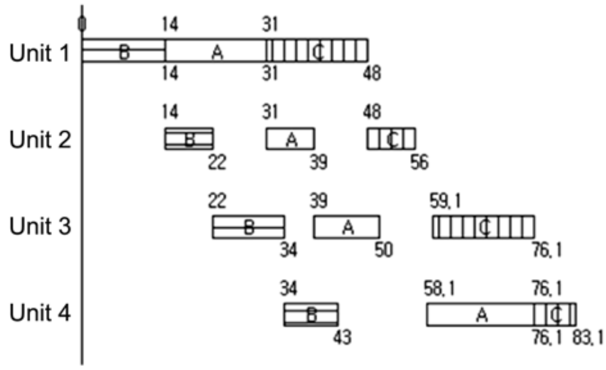


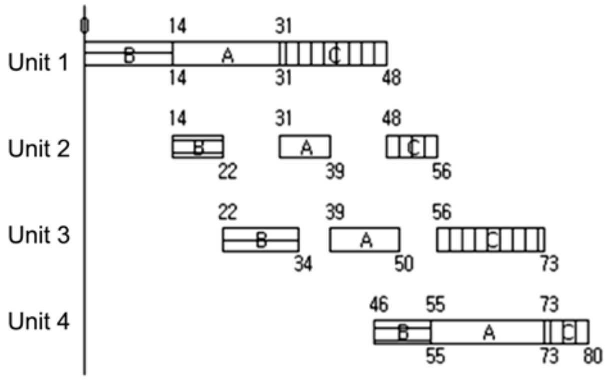
Fig. 4. Result of Example 1-2: Distributions of makespan with different objective function (unit: Hr).



(a) Expected makespan



(b) Expected makespan plus variance



(c) Expected makespan plus standard deviation

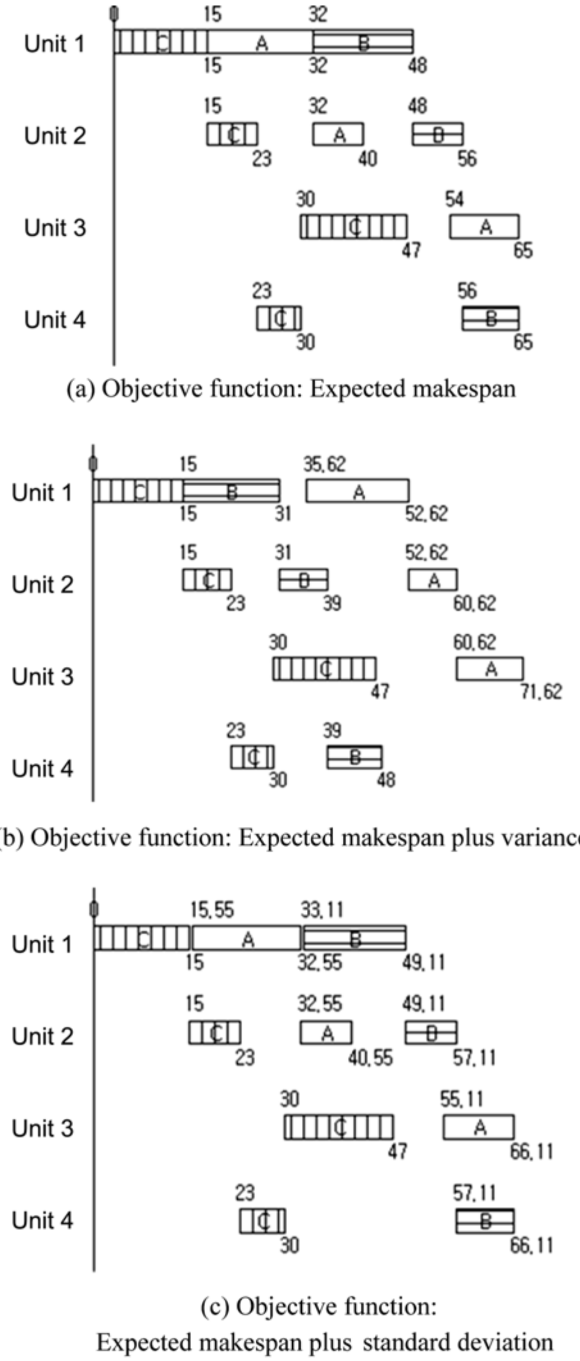
Fig. 5. Gantt charts based upon different objective functions for the case of Example 1-1.

problems representing process operation decision-making practices under the presence of uncertainty. As the effect of multiple scenarios, different results were obtained. As we sorted out their distribution numerically, their shape of distribution can be graphically illustrated as in Fig. 3 and 4. Here it is right that the sum of distributed factors is 1. But the actual shape of the distribution can be different depending on the terms in the objective function.

Second, we can figure out that the process can be finished between 75 and 92.5 hr. Representative Gantt charts are shown in Figs. 5 and 6 with processing times of highest probability. Numerical values of expected values and variances are summarized in Table 2.

Table 2. The values of objective function and metric, Example 1

Problem	Objective type	Characteristics	
		Expectation (hr)	Variance
Example 1-1	E (MS)	81.08	11.87
	E (MS)+V (MS)	83.60	1.18
	E (MS)+σ (MS)	81.85	5.08
Example 1-2	E (MS)	68.85	13.18
	E (MS)+V (MS)	72.12	1.26
	E (MS)+σ (MS)	69.25	9.86



(a) Objective function: Expected makespan

(b) Objective function: Expected makespan plus variance

(c) Objective function: Expected makespan plus standard deviation

Fig. 6. Gantt charts based upon different objective functions for the case of Example 1-2.

Third, there can be a tradeoff between the maximum performance and robustness to variations. To maximize the performance, more attention should be given to minimize the makespan. On the other hand, to compute an operation plan insensitive to the variation, robustness should be given more priority. This trade-off can be reflected in the objective function as in the following format.

$$(1-\alpha)\cdot\sum_s \mathrm{P}_s\cdot \mathrm{MS}_s+\alpha\cdot\sqrt{\sum_s \mathrm{P}_s\cdot \mathrm{MS}_s^2-\sum_s(\mathrm{P}_s\cdot \mathrm{MS}_s)^2},\quad 0\le\alpha\le 1 \quad (5c)$$

$$(1-\alpha)\cdot\sum_s \mathrm{P}_s\cdot \mathrm{MS}_s+\alpha\cdot\left(\sum_s \mathrm{P}_s\cdot \mathrm{MS}_s^2-\sum_s(\mathrm{P}_s\cdot \mathrm{MS}_s)^2\right),\quad 0\le\alpha\le 1 \quad (5d)$$

Fourth, uncertain parameters are considered in discrete distribution in this paper. If it is a continuous distribution, we integrate them analytically or by quadrature methods in order to treat them mathematically. If it is a discrete distribution, calculating steps is simple with objective values multiplied by their probability. However, integrating procedure of continuous distributions leads to severe nonlinearity. Mathematical optimization programs cannot solve large-sized problems. A perfect continuous distribution is built only after sampling and analyzing of infinite data sets. Instead, these distributions are generally made by estimating a suitable sample size. In comparison to the complexity given by continuous ones with loose approximation, the mathematical tackling effort has no internal rate of return. Reasonable approximation to a discrete distribution would not rather distort the procedure.

So far, short-term decision-making problems considering the presence of uncertainty have been investigated by using the scenario-based scheme. In the next, longer-term planning problems will be addressed.

SCENARIO-BASED ROBUST LONG-TERM OPERATION PLANNING APPROACH

This section will address how a scenario-based scheme can handle macro scale process operation problems of long time horizon. As an alternative illustration, polyvinyl chloride (PVC) manufacturing supply chain problems will be presented.

Recently, process systems engineering has been actively investigating supply chain and its management. Various types of studies have been made such as reviews [25,42], simulation framework [26,27]. But most of the works approach the problem by formulating operation planning of supply chain networks as a mathematical optimization problem. There are some works which expand the area of interest by incorporating SCM with other decision management

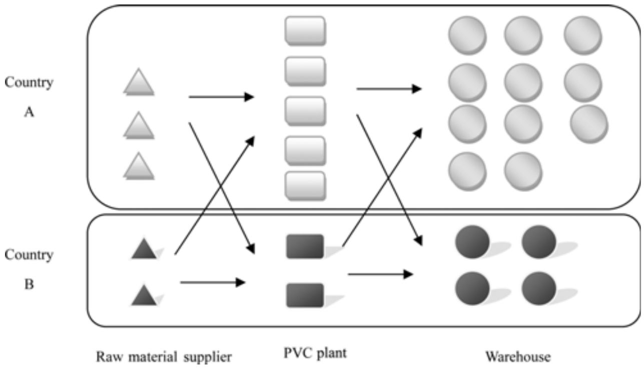


Fig. 7. Illustrative PVC supply chain.

such as asset and capital management [28] or scheduling [17,29], planning [30,35].

PVC is one of the largest commodity chemicals and a general purposed plastic material consumed in great quantities around the world. PVC is mainly classified into four kinds depending upon straight or paste and special or normal. In PVC manufacturing supply chains, there are mainly three levels: raw material suppliers that are ethylene plants, PVC manufacturing plants, and warehouses in the local markets. In this case, there are five ethylene plants that are raw material suppliers for PVC, seven PVC plants and fifteen warehouses. It is assumed that demands of warehouses are to be sold. As can be seen in Fig. 7, there are two countries A and B. There are three suppliers, five plants of PVC and eleven warehouses in country A and two suppliers, two plants and four warehouses in country B.

It is assumed that the amount of material from the ethylene supplier is assumed to be unlimited and capacities of plants are known and fixed. The prices of PVC can be different with regard to their quantity. Transportation costs are between raw material supplier and plant and between plant and warehouse. Inventory costs and production costs in plants are given. Demands of each warehouse with respect to 12 months are also known.

1. Deterministic Problem: Example 2-1

Supply chain problems should determine the following decisions: (1) how much to produce at individual PVC plants, and (2) how much to transport them into which warehouses, and (3) how much to store at each warehouse for inventory to meet the demand. For example, the demand of week 1 denoted as W01 in country A, one of 15 warehouses, is graphically illustrated in Fig. 8.

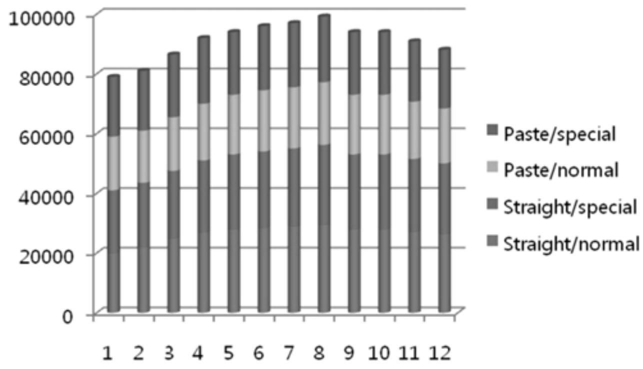


Fig. 8. Graphical illustration of demand fluctuation over 12 months (x-axis).

A supply chain decision-making model involves basic mass balances between raw material suppliers, manufacturing plants and warehouses including capacity constraints being set. Here, a quantity discount as used by Tsiakis et al. [31] is applied to reflect the practical situation. For example, when continuous variable Q is divided into NR regions, the cost in terms of quantity increase can be formulated into the following set of relationships.

$$\mathbf{Z}_d=\begin{cases}\mathbf{1},\ \text{if}\ \mathbf{Q}\in[\overline{\mathbf{Q}}_{d-1},\ \overline{\mathbf{Q}}_d] \\ \mathbf{0},\ \text{otherwise}\end{cases} \tag{16}$$

$$\overline{\mathbf{Q}}_{d-1}\mathbf{Z}_d\le\mathbf{Q}_d\le\overline{\mathbf{Q}}_d\mathbf{Z}_d \tag{17}$$

$$\sum_{d=1}^{NR}\mathbf{Z}_d=\mathbf{1} \tag{18}$$

$$\mathbf{Q}=\sum_{d=1}^{NR}\mathbf{Q}_d \tag{19}$$

$$\mathbf{C}=\sum_{d=1}^{NR}\left[\overline{\mathbf{C}}_{d-1}\mathbf{Z}_d+(\mathbf{Q}_d-\overline{\mathbf{Q}}_{d-1}\mathbf{Z}_d)\frac{\overline{\mathbf{C}}_d-\overline{\mathbf{C}}_{d-1}}{\overline{\mathbf{Q}}_d-\overline{\mathbf{Q}}_{d-1}}\right] \tag{20}$$

The objective value is to maximize the profit, which is the difference between revenue and cost:

$$\text{Profit=Revenue} \\ -(\text{transportation cost+inventory cost+operating cost}) \tag{21}$$

The profit can be formulated into the following equation, and this work employs the standard deviation of the profit as the objective function.

$$\begin{aligned}\mathbf{Profit}=\sum_d\Bigg\{&\sum_{p,w,m}\mathbf{Prl}_{p,l}\mathbf{qk}_{k-1}\mathbf{Z}_{p,w,l,m,d}\\&+(\mathbf{Q}_{p,w,l,m,d}-\mathbf{qk}_{k-1}\mathbf{Z}_{p,w,l,m,d})\frac{\mathbf{Prl}_{p,l}\mathbf{qk}_d-\mathbf{Prl}_{p,l}\mathbf{qk}_{d-1}}{\mathbf{qk}_d-\mathbf{qk}_{d-1}}\Bigg\}\\&-\Bigg\{\sum_{r,p,m}\mathbf{Trri}_{r,p}\mathbf{X}_{r,p,m}+\sum_{p,j,l,m}\mathbf{Trij}_{p,w}\mathbf{y}_{p,w,l,m}\\&+\sum_{p,l,m>1}\mathbf{civ}_p\mathbf{IV}_{p,l,m-1}+\sum_{p,l,m}\mathbf{Opi}_{p,l}\mathbf{PA}_{p,l,m}\Bigg\}\end{aligned} \tag{22}$$

The above objective function is subject to the following constraints: At first, the production amount is limited by its lower and upper bound:

$$\mathbf{CP}_{p,m}^{Min}\le\sum_l\mathbf{PA}_{p,m,l}\le\mathbf{CP}_{p,m}^{Max}\quad\forall\mathbf{p},\mathbf{m} \tag{23}$$

Eq. (24) denotes inventory balance relations.

$$\mathbf{IV}_{p,m,l}=\mathbf{IV}_{p,m-1,l}+\mathbf{PA}_{p,m,l}-\sum_w\mathbf{Y}_{p,w,m,l} \tag{24}$$

Production quantity is dependent upon raw material.

$$\sum_r\mathbf{X}_{r,p,m}=\sum_l\mathbf{PA}_{p,m,l}\quad\forall\mathbf{p},\mathbf{m} \tag{25}$$

A warehouse should possess more amount than its demand.

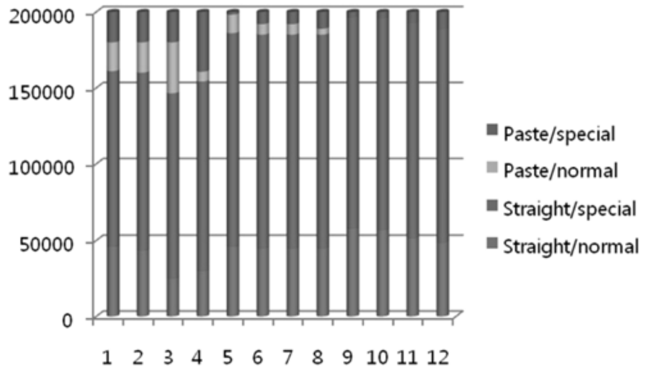


Fig. 9. Amount of production at plant P1.

$$\sum_p \mathbf{Y}_{p,w,m,l} \geq \mathbf{DJ}_{w,l,m} \quad \forall \mathbf{w}, l, \mathbf{m} \tag{26}$$

Product amounts for sale at each warehouse that should be transported from plants.

$$\mathbf{Y}_{p,w,m,l} = \sum_d \mathbf{Q}_{p,w,m,l,d} \quad \forall \mathbf{p}, \mathbf{w}, \mathbf{m}, l \tag{27}$$

Only one section is selected to decide the amount.

$$\sum_d \mathbf{Z}_{p,w,m,l,d} = 1 \quad \forall \mathbf{p}, \mathbf{w}, \mathbf{m}, l \tag{28}$$

The sales amount is divided into multiple regions to compute the discrete values.

$$\mathbf{qk}_{d-1}\mathbf{Z}_{p,w,m,l,d} \leq \mathbf{Q}_{p,w,m,l,d} \leq \mathbf{qk}_d\mathbf{Z}_{p,w,m,l,d} \tag{29}$$

In case of plant P1, one of the seven is presented in Fig. 9, including production and transportation with respect to each month as shown in Table 3.

The solution of the problem based on the proposed mathematical model is computed by using CPLEX 7.0.0 in a Pentium III 1,000 MHz PC. The computational time is 7.3 seconds CPU time. The model involves 16,212 continuous variables, 10,080 binary variables and 26,424 constraints.

2. Uncertain Problem: Example 2-2

A stochastic programming problem is formulated to consider the presence of variation. Particularly, the variation in transportation cost due to the fluctuation of crude oil price is taken into account. The oil price has been fluctuating continuously and is expected to continue. In this work, the oil price fluctuation between 2002 and 2004 has been discretized with its mean value as summarized in Fig. 10 and Table 4. The objective function based on multiple scenarios is as follows:

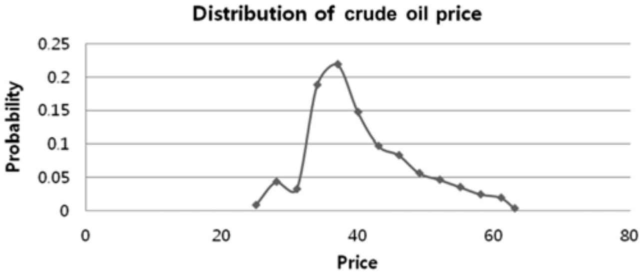


Fig. 10. Scenario of crude oil price ($bbl).

Table 4. Distribution of probability of oil price ($/bbl)

Price	Probability	Price	Probability
25	0.008	46	0.083
28	0.043	49	0.056
31	0.032	52	0.046
34	0.188	55	0.035
37	0.219	58	0.024
40	0.147	61	0.019
43	0.097	63	0.003

$$\begin{aligned}\mathbf{Profit} = \sum_s \mathbf{p}_s \Bigg[\sum_{d>1} \Bigg\{ &\sum_{p,w,m} \mathbf{Prl}_{p,l,s}\mathbf{qk}_{d-1}\mathbf{Z}_{p,w,l,m,d} \\ &+ (\mathbf{Q}_{p,w,l,m,d} - \mathbf{qk}_{d-1}\mathbf{Z}_{p,w,l,m,d}) \frac{\mathbf{Prl}_{p,l}\mathbf{qk}_d - \mathbf{Prl}_{p,l}\mathbf{qk}_{d-1}}{\mathbf{qk}_d - \mathbf{qk}_{d-1}} \Bigg\} \\ &- \Bigg\{ \sum_{r,p,m} \mathbf{Trri}_{r,p,s}\mathbf{X}_{r,p,m} + \sum_{p,w,l,m} \mathbf{Trij}_{p,w,s}\mathbf{y}_{p,w,l,m} \\ &+ \sum_{p,l,m>1} \mathbf{civ}_p\mathbf{IV}_{p,l,m-1} + \sum_{p,l,m} \mathbf{Opi}_{p,l,s}\mathbf{PA}_{p,l,m} \Bigg\} \Bigg]\end{aligned} \tag{30}$$

Table 3. Amount of transportation from plant P1 to warehouses

	1	2	3	4	5	6	7	8	9	10	11	12
W01	41000	41500	42500	44000	26000	27000	27500	28700	26000	26000	24500	23600
W02	0	0	1000	4500	6000	7000	8000	9200	6000	6000	4500	3600
W03	0	0	0	0	4000	4900	5900	7600	13000	13000	11500	10000
W04	10000	10000	11300	12000	43000	45300	46700	48500	15100	14000	15600	27700
W05	10000	13550	27500	32000	35000	33500	20000	20000	12250	16000	20000	20000
W06	36000	39000	42500	42500	51000	51000	51000	52500	54000	54000	51900	51300
W07	29500	29500	32850	33000	37000	40000	40000	44000	48600	46100	46100	42800
W08	56500	62000	64000	66000	70000	70000	70000	71500	71500	70000	68500	67000
W09	0	0	2800	4500	8000	9000	9000	10450	10450	10450	6600	3600
W10	0	0	0	0	0	1000	2000	2000	1600	1200	800	400
W11	0	0	0	0	0	0	0	0	0	0	0	0
W12	0	0	0	0	0	0	0	0	0	0	0	0

Based upon the variations of oil price, other parameters are also subject to variation. Their variation forms can be determined as follows. According to data from 2002 to 2004, 2.4% of diesel price changed by the change of 1\$/bbl crude oil. This effect could be taken as one-fifth when the controlling policy relaxes the fluctuation of fuel oil. Transportation cost of product is assumed to be affected by 1.8%, which consists of the main wage.

$$\mathbf{Trij}_{p,w,s}=\mathbf{Trij}_{p,w}\times(1+0.0.18(\mathbf{PCO}_s-\mathbf{MM})/5)\quad \forall \mathbf{p, w, s} \tag{31}$$

where MM denotes the mean crude oil price and PCO_s is the price of crude oil at scenario s.

Transportation cost of resource could be affected by 3% and one-third effect, which is weakly controlled by tax.

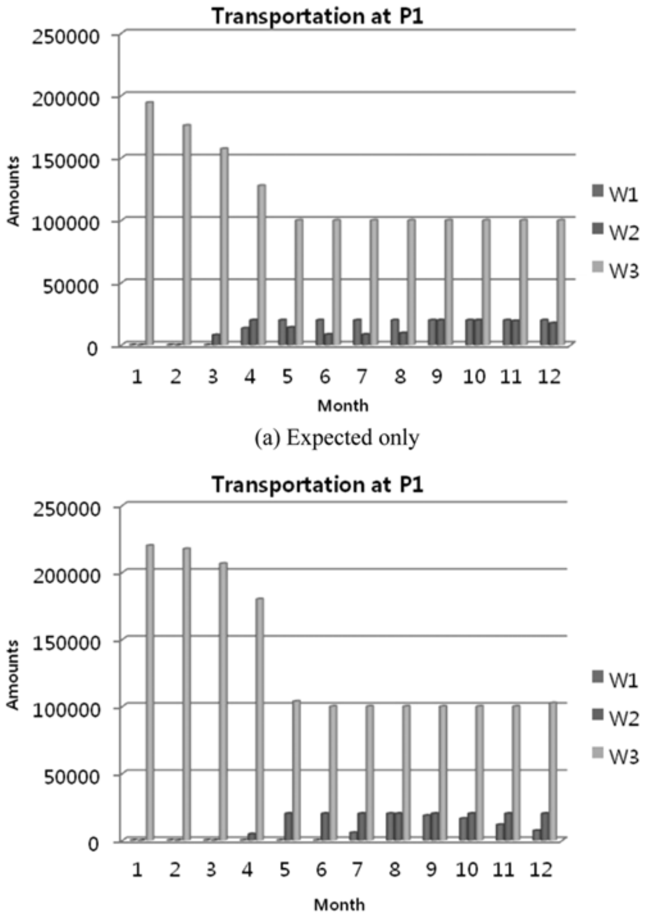
$$\mathbf{Trri}_{r,p,s}=\mathbf{Trri}_{r,p}\times(1+0.03(\mathbf{PCOs}-\mathbf{MM})/3)\quad \forall \mathbf{p, r, s} \tag{32}$$

Product price and operating cost are assumed to be affected by 0.8% and 0.5%, respectively.

$$\mathbf{prl}_{p,l,s}=\mathbf{prl}_{p,l}\times(1+0.008(\mathbf{PCO}_s-\mathbf{MM}))\quad \forall \mathbf{p}, \boldsymbol{l}, \mathbf{s} \tag{33}$$

$$\mathbf{Opi}_{p,l,s}=\mathbf{Opi}_{p,l}\times(1+0.005(\mathbf{PCO}_s-\mathbf{MM}))\quad \forall \mathbf{p}, \boldsymbol{l}, \mathbf{s} \tag{34}$$

A stochastic planning problem involving the above constraints with oil price scenarios based on the data in the previous example is computed. We made four price discount points and fourteen scenarios. The proposed framework has been employed with different objective functions. As an illustration, a part of the solution is graphically described in Fig. 11 and summarized in Table 5.



(a) Expected only

(b) Robustness (α=5/6)

Fig. 11. Result of plant P1 (W1, W2, W3 denote different warehouse).

Table 5. Values of objective function and metric, example 2-2

Objective type	Expected value (\$)	Standard deviation
E(Profit)	7,072 M	5.86e+8
(1/2)E(Profit)−(1/2)s(Profit)	7,071 M	5.86e+8
(1/6)E(Profit)−(5/6)s(Profit)	7,062 M	5.83e+8
(1/9)E(Profit)−(8/9)s(Profit)	7,033 M	5.79e+8

CONCLUDING REMARKS

There are a few issues for further comment on this work. First, the proposed model cannot answer question such as what the price is next week. However, it can be used to establish decision-making frameworks, for example, home-delivery logistic companies to make one-year-contracts with shopping companies. The proposed scenario-based scheme could provide more insights like the best fee policy against a varying environment like oil price fluctuations.

Second, this paper is concerned with computing the expected value of the objective as well as estimating its distribution in response to the variation to measure the robustness instead of focusing on computing the expected one. In process operation in the presence of uncertainty, there could be many types to measure the degree of robustness. This paper employed the standard deviation of the expected makespan and cost.

Third, it would be interesting to diversify the types of uncertainty in a more categorized manner, such as local and globally affecting parameter variation, temporary or continuous ones, simultaneous or step-by-step ones, etc. Fourth, this work used historic data as scenarios, but future events can be also taken into account. The proposed scenario scheme can be employed to address problems for new products or processes without real data. The potential expected parameters with their probabilities can be set as parameters.

Fourth, it is true that the scheduling example is not detailed compared to the SCM example. Two types of multiple product manufacturing process types, multiproduct batch process and multipurpose batch process, are considered in this paper. Unfortunately, it was not possible to obtain the actual industrial case including the data in the scheduling problem. We think the presented 'simply' looking example fully conveys the message this paper aims.

This paper discussed mathematical approaches to solve process operations problems under variation in an effort to apply them to some real situations. Given that the information is subject to being wrong, robust tools and methodologies should be continuously searched and developed.

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NOMENCLATURES

Sets

I : set of sequence (i=1, …, I)
J : set of units (j=1, …, J)
K : set of products (k=1, …, K)
NR : set of discrete points (d=1, …, NR)
L : set of items (l=1, …, L)
M : set of months (m=1, …, 12)
P : set of plants (p=1, …, P)
R : set of resources (r=1, …, R)
S : set of scenario (s=1, …, S)
W : set of warehouses (w=1, …, W)

Indices

k : products
j : units
j(k) : units in which process product k
j′(k, j) : units in which process product k directly before unit j
i : sequence
i(j) : sequence in unit j
s : scenario
d : discrete point

Parameters

α : weighting factor for robustness
$\overline{\mathbf{C}}_d$: cost as a parameter at discrete point d
MM : mean value of crude oil price
M : sufficiently large positive number
NR : number of discrete point d
$Op_{p,s}$, Opi_p : operating cost at plant p in scenario s
$prl_{p,s}$, Pr_p : price of product of plant p in scenario s
$PT_{k,j,s}$: processing time of product k in unit j at scenario s
P_s : probability to scenario s
PCO_s : price of crude oil at scenario s
$\overline{\mathbf{Q}}_d$: quantity as a parameter at discrete point d
$Tij_{p,w}$, $Tr_{p,w,s}$: transportation cost from plant p to warehouse w in scenario s
$Tri_{r,p}$, $Trr_{r,p,s}$: transportation cost from supplier r to plant p in scenario s

Variables

C : cost
$C_{i,j,s}$: completion time of sequence i in unit j at scenario s
$C_{k,j,s}$: completion time of product k in unit j at scenario s
MS_s : makespan at scenario s
$Q_{p,w,l,m,d}$: sales amount from plant p to warehouse w of item l at time m
$w_{k,i}$: binary variable, valued 1 when product k process ith sequence
$w_{k,i,j}$: binary variable, valued 1 when product k process ith sequence in unit j
Z_d, $Z_{p,w,l,m,d}$: binary variable for quantity discount, valued 1 when discrete point d is selected in
$X_{r,p,m}$: transportation amount from raw material suppler r to plant p
$y_{p,w,l,m}$: transportation amount from plant p to warehouse w of item l at time m
$PA_{p,l,m}$: production amount of item l in plant p in time m
$IV_{p,l,m}$: inventory level of item l in plant p in time m
$Q_{p,w,l,m,k}$: sales amount of item l in plant p in time m

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