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Transient responses of two-dimensional viscoelastic film casting processes to sinusoidal disturbances

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Abstract−The sensitivity of two-dimensional isothermal film casting processes to ongoing sinusoidal disturbances has been investigated by using the frequency response method with transient simulation techniques. Amplitude ratios of state variables such as cross-sectional area, film width and film thickness at the take-up position with respect to a sinusoidal disturbance show resonant peaks along the frequency domain. Effects of operating conditions, such as drawdown ratio and aspect ratio, on the process sensitivity have been examined. Increasing drawdown ratio and decreasing aspect ratio make the system more sensitive to disturbances. Also, the dichotomous behavior in the sensitivity analysis using viscoelastic Phan-Thien and Tanner fluids has been elucidated. This frequency response method can be a useful tool to optimally design process systems for better processability and product quality.

Key words: Sensitivity, Film Casting, Frequency Response, Transient Simulation, Sinusoidal Disturbance

INTRODUCTION

The film casting process is an industrially important polymer extensional deformation process to manufacture many kinds of thin films [1]. The molten polymer is extruded from a slit die and stretched down to the rotating chill or take-up roll as depicted in Fig. 1. Recently, the rapid growth of IT cutting-edge technologies has augmented the need for advanced quality control of film products, such as optical compensation films which are superior in clarity and optical anisotropy for display devices. Enhancing the process productivity and the uniformity of the film products in this process always entails an in-depth understanding of the dynamics and stability of the system. As in any other industrial processes, various kinds of unexpected disturbances inevitably affect the process stability and sensitivity, giving rise to serious defects of produced films.

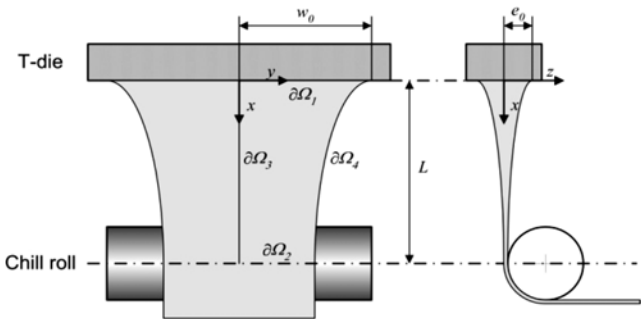

Fig. 1. The schematic diagram of film casting process.

There exist, of course, instabilities and defects in film casting processes: draw resonance, neck-in of the film width, and edge bead on the final film product [1,2]. Draw resonance instability, which is distinguished by periodic oscillations of state variables such as film thickness, film width, and tension, arises as the drawdown ratio (ratio of fluid velocities at take-up roll and slit die) is raised beyond its onset [3-6]. The same phenomenon can be also manifested in other extensional deformation processes, e.g., fiber spinning and film blowing [2,7]. Neck-in of film width along the machine direction due to the strong extensional deformation in the drawdown region from the die to take-up roll and edge bead (or dog-bone) with thicker film thickness at edges than at center is not uncommon in the film casting process.

Since the first works laid the foundation for studying film casting [8,9], exploiting the draw resonance instability of this process, there have been many research efforts on the dynamics and stability in this process. Co's group interestingly presented the stability results with more realistic viscoelastic fluids [10,11] using constant 1-D models. Agassant's group developed varying width 1-D models to explain the neck-in defect and also predicted both edge bead and neck-in as well as draw resonance using 2-D models [12-15]. Hyun's group [4,5], for the first time, calculated the transient behavior of film profiles in 2-D isothermal/nonisothermal film casting processes with viscoelastic fluids, e.g., upper-convected Maxwell and Phan-Thien and Tanner fluids, confirming good agreement with experimental observations.

Also, a sensitivity analysis, as well as the conventional stability one, is indispensable to ensure optimal process conditions for the film uniformity. In real manufacturing processes, even in well-designed ones, they are invariably subject to any small ongoing disturbances such as periodic oscillation of take-up roll speed, flow rate, etc. For this reason, the sensitivity of processes has typically been

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‡This article is dedicated to Professor Chul Soo Lee in commemoration of his retirement from Department of Chemical and Biological Engineering of Korea University.

examined from the response to sinusoidal disturbances with various frequency spectra, called frequency response analysis. Jung et al. [16,17] applied the frequency response method in the melt spinning processes, and Lee et al. [18] extended this method to 1-D film casting models. However, as yet, the sensitivity analysis in 2-D film casting with viscoelastic models has not been explored.

In this study, transient simulation with ongoing sinusoidal input disturbances has been carried out to scrutinize the sensitivity of the 2-D isothermal film casting process using up-to-date FEM numerical techniques. Results by this numerical scheme for solving the full set of partial differential governing equations are analogous to the frequency response method using linearized perturbation equations of the system. With this method, essential information about the sensitivity of the process will be successfully acquired.

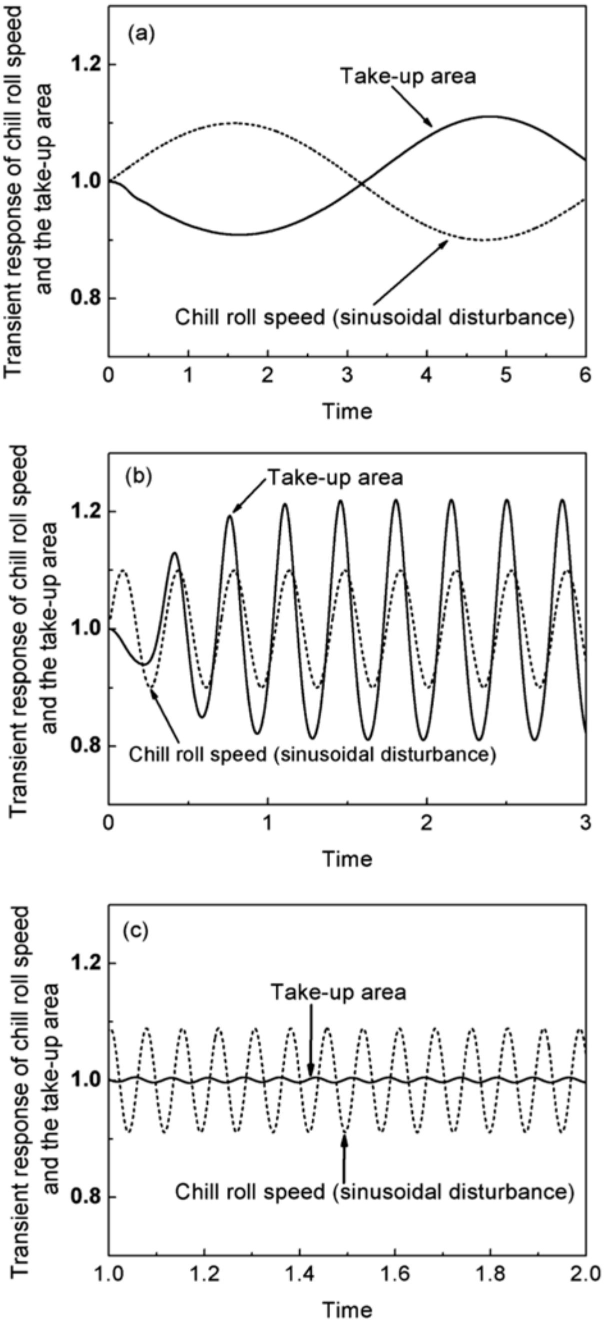

Fig. 2. Transient responses of the take-up area where sinusoidal perturbation at chill roll speed is introduced under various frequencies (a) ω=1, (b) ω=18, and (c) ω=83 (A_r=0.6, D_r=10, De=0.005, ε=0.015, ξ=0.1).

TIME-DEPENDENT GOVERNING EQUATIONS

The dimensionless governing equations for 2-D isothermal film casting system are expressed below based on the models by Silagy et al. [12] and Kim et al. [4]. The Phan-Thien Tanner constitutive equation, which is well known for delineating the extensional behavior of polymeric liquids, is incorporated here for describing the viscoelastic nature [5,19].

Equation of continuity: $\dfrac{\partial e}{\partial t} + \nabla \cdot e\mathbf{v} = 0.$ (1)

where, $e = \dfrac{\bar{e}}{\bar{e}_0}$, $\mathbf{v} = \dfrac{\bar{\mathbf{v}}}{\bar{v}_0}$, $t = \dfrac{\bar{t}\bar{v}_0}{\bar{w}_0}$, $\nabla = \bar{w}_0 \bar{\nabla}$.

Equation of motion: $\nabla \cdot e\sigma = 0.$ (2)

where, $\sigma = \dfrac{\bar{\sigma}\bar{w}_0}{\eta_0 \bar{v}_0}$

Constitutive equation (PTT model):

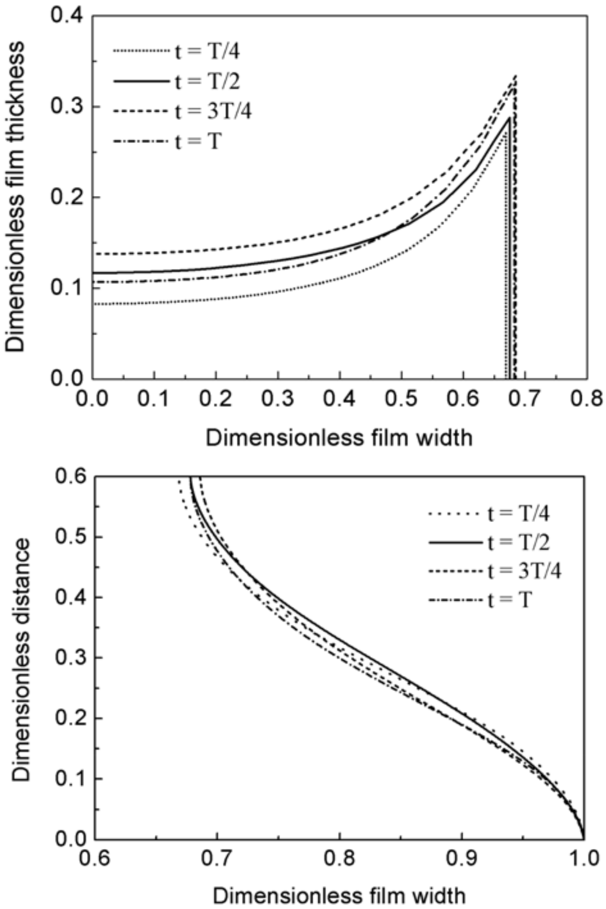

Fig. 3. One-cycle evolution of the film thickness at take-up and the film width due to the sinusoidal disturbance at take-up speed (A_r=0.6, D_r=10, De=0.005, ε=0.015, ξ=0.1, ω=18, T=period of oscillation).

$$K\tau + De\left[\frac{\partial\tau}{\partial t} + \mathbf{v}\cdot\nabla\tau - \mathbf{L}\cdot\tau - \tau\cdot\mathbf{L}^T\right] = 2\mathbf{D}. \qquad (3)$$

where, $\tau = \frac{\tau \bar{w}_0}{\eta \bar{v}_0}$, $K = \exp[\varepsilon De\, tr\,\tau]$, $De = \frac{\lambda \bar{v}_0}{\bar{w}_0}$, $\mathbf{L} = \nabla\mathbf{v} - \xi\mathbf{D}$,

$$2\mathbf{D} = [\nabla\mathbf{v} + (\nabla\mathbf{v})^T]$$

Boundary conditions:

(i) Inlet ($\partial\Omega_1$): $v_x=1, v_y=0, e=1, w=1, \tau=\tau_0$ at t=0. (4a)

$v_x=1, v_y=0, e=1, w=1$ at t>0. (4b)

(ii) Outlet ($\partial\Omega_2$): $v_x = D_r \equiv \frac{\bar{v}_L}{\bar{v}_0}$, $v_y = 0$ at t=0. (4c)

$v_x = D_r(1+\delta\sin(\omega t))$, $v_y=0$ at t>0. (4d)

(iii) Center ($\partial\Omega_3$): $\sigma_{xy}=0$ at t≥0. (4e)

(iv) Edge ($\partial\Omega_4$): $\frac{\partial w}{\partial t} + v_x\frac{\partial w}{\partial x} = v_y$, $\sigma\cdot\mathbf{n} = 0$ at t≥0. (4f)

The boundary at the center position represents a symmetric condition. To implement periodic perturbation in the system, an ongoing sinusoidal change (or disturbance) on the chill roll speed (Eq. (4d)) has been applied to the governing equations, leading to transient sinusoidal response of the film thickness, film width and take-up area. Also, such a sinusoidal disturbance like Eq. (4d) can be imposed to the inlet boundary conditions to account for the sensitivity result influenced by the perturbation at extrusion speed. Several assumptions are adopted in this model. Isotropic pressure is equal to the normal stress (τ_{zz}) and crystallization, extrudate swell and secondary forces are not considered.

The same numerical finite element methods (FEMs) and schemes developed by Kim et al. [4] and Shin et al. [5] have been used for obtaining the amplitude ratio of state variables such as film thickness, film width, and take-up area: Spine and Arbitrary Lagrangian Eulerian (ALE) algorithm for free surface tracking, streamline upwinding/Petrov Galerkin (SU/PG) method for stabilization technique, and 2^{nd}-order Gear method for time discretization. More detailed numerical methods are referred to Kim et al. [4].

RESULTS AND DISCUSSION

The ongoing disturbance at take-up speed has been introduced to the base or steady-state flow with 10% sinusoidal variation in frequency domain. In Fig. 2, the corresponding transient responses to the oscillation of the take-up area are depicted under the given frequencies. The take-up area was evaluated from film width and film thickness data at the take-up using typical numerical integration method, e.g., Simpson's rule. It can be shown that the sine-wave curve of an output variable is obtained along with the evolution of time and the amplitude of transient output changes with the frequency. The amplitude ratio of output to input curves stands for the sensitivity of the system which is relying on the frequency. The higher amplitude ratio of the response brings about a more highly sensitive system. This amplitude ratio with respect to the ongoing sinusoidal disturbance shows resonance peaks along the frequency regime, demonstrating that the film casting system is hyperbolic [16,20]. Effects of operating conditions such as drawdown ratio (D_r), aspect ratio (A_r) and material parameter like Deborah number (De) embedded in fluid viscoelasticity on the sensitivity have also been examined by introducing a sinusoidal disturbance at take-up speed. It is noted that transient responses of state variables with respect to

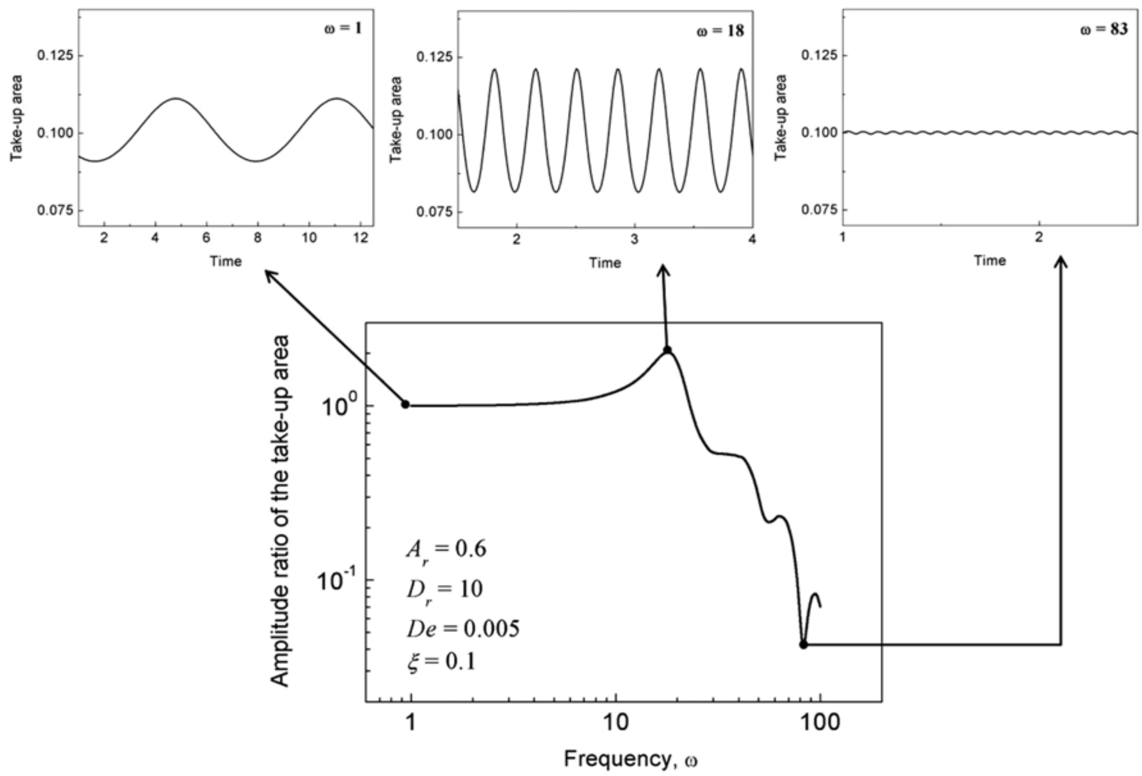

Fig. 4. Frequency response of the take-up area where sinusoidal perturbation at chill roll speed is introduced under various frequencies (A_r=0.6, D_r=10, De=0.005, ε=0.015, ξ=0.1).

the sinusoidal disturbance at extrusion speed are almost the same as those by the sinusoidal disturbance at take-up speed.

1. Sensitivity of the Film Dimension

The key feature in the sensitivity analysis using the 2-D film casting system, compared with the 1-D case, is that sensitivities of both film width and film thickness, which varies with film width direction, due to the edge-bead phenomenon can be clearly probed (e.g., Figs. 3 and 5). The 1-D case only predicted the constant film thickness in the take-up region [18]. In order to observe the overall effect on the sensitivity of final film products, however, the sensitivity of the take-up area will be mainly focused on by using the transient data of film width and film thickness along with width direction at the take-up. Fig. 3 displays the one-cycle evolution of the film thickness at take-up position and the film width when a sinusoidal disturbance with frequency, ω=18 is introduced at take-up speed.

Fig. 4 shows an example of a bode diagram, as frequently presented in control theory, which accounts for the amplitude ratio of a state variable, e.g., take-up area here, along with frequency regime from transient simulation data. The amplitude ratio of the take-up area (also, film thickness and film width) shows the resonant peaks along with frequency. At the low frequency region, the amplitude ratio has unity value when a perturbation changing mass flow rate is introduced, as described in previous literatures [16,18].

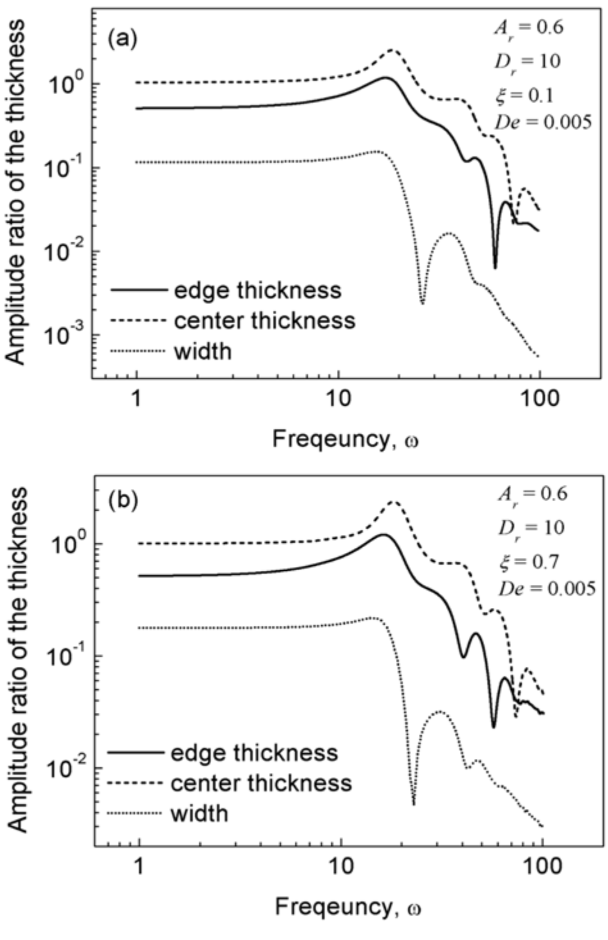

Fig. 5. Frequency response of film width and film thicknesses at edges and center for (a) extension-thickening fluid (ε=0.015, ξ=0.1) and (b) extension-thinning fluid (ε=0.015, ξ=0.7).

As mentioned before, the 2-D model considerably predicts the edge bead phenomenon so that it can give more realistic information for the sensitivity difference between film thicknesses at the center and edges, as shown in Fig. 5. The sensitivity of the center thickness is rather higher than that of the edge thickness, which may be attributed to the different thickness size at steady state and different extensional deformation type, i.e., biaxial extension at center and uniaxial extension in edges. Also, the sensitivity of film width is one order less than the others. Accordingly, the film thickness is more sensitive than film width with respect to the variation of the take-up speed.

2. Effect of Operating Conditions: Drawdown Ratio and Aspect Ratio

Effects of drawdown ratio (D_r) and aspect ratio (A_r) on the sensitivity are presented in Fig. 6 in which an ongoing sinusoidal disturbance has been imposed to the take-up speed. It has been found that as the drawdown ratio increases, the system becomes more sensitive showing higher amplitude ratio of the take-up area. However, increasing the aspect ratio, within a typical operating range from 0.2 to 1.2, results in a less sensitive system to disturbances.

3. Effect of Fluid Viscoelasticity: Deborah Number

Figs. 7 and 8 portray the frequency response results in the case of different fluid viscoelasticities. In extension-thinning fluids (Fig. 7), the amplitude ratio of the take-up area rises with increasing De. In

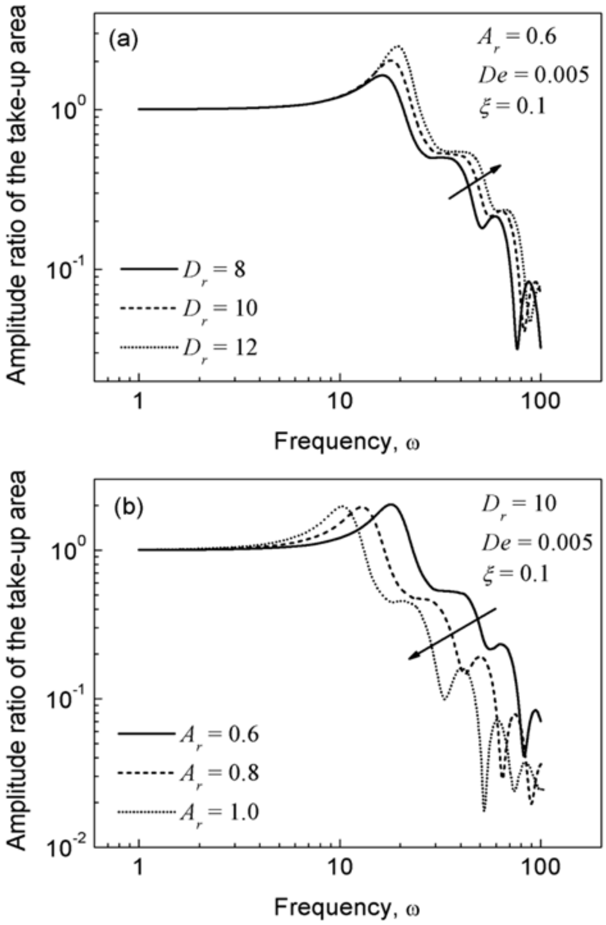

Fig. 6. Effects of (a) drawdown ratio and (b) aspect ratio on the sensitivity.

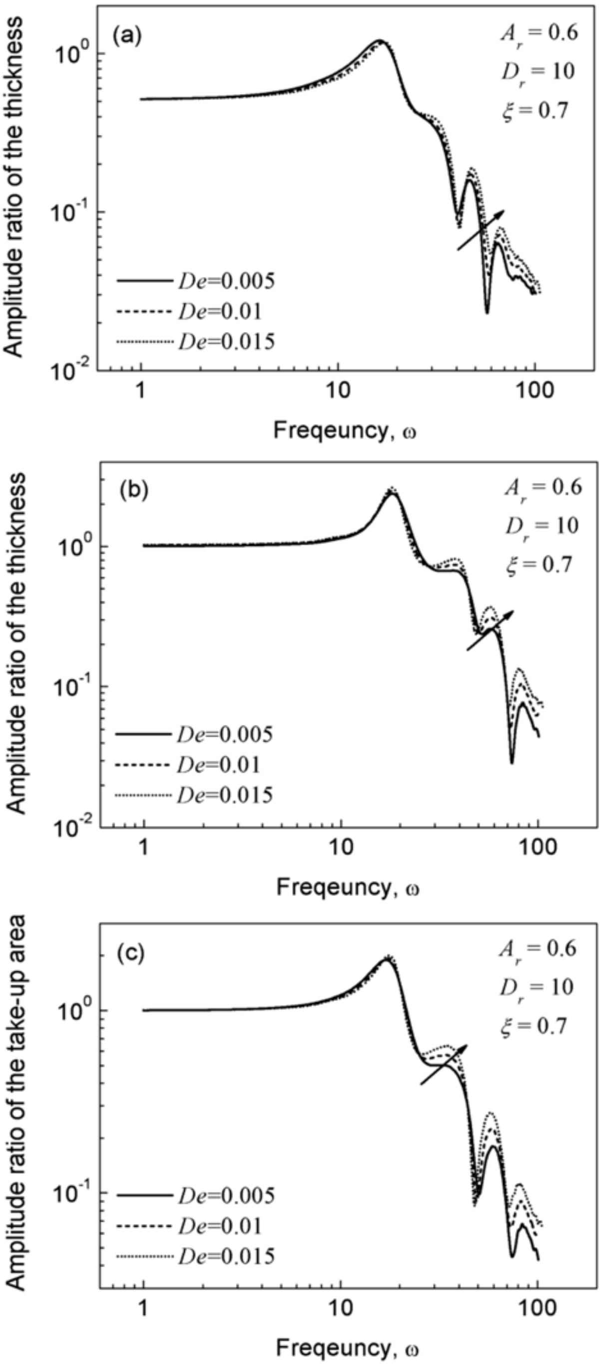

Fig. 7. Frequency response of (a) film thickness at edges, (b) film thickness at center, (c) take-up area for extension-thinning fluids with different Deborah numbers (ε=0.015, ξ=0.7).

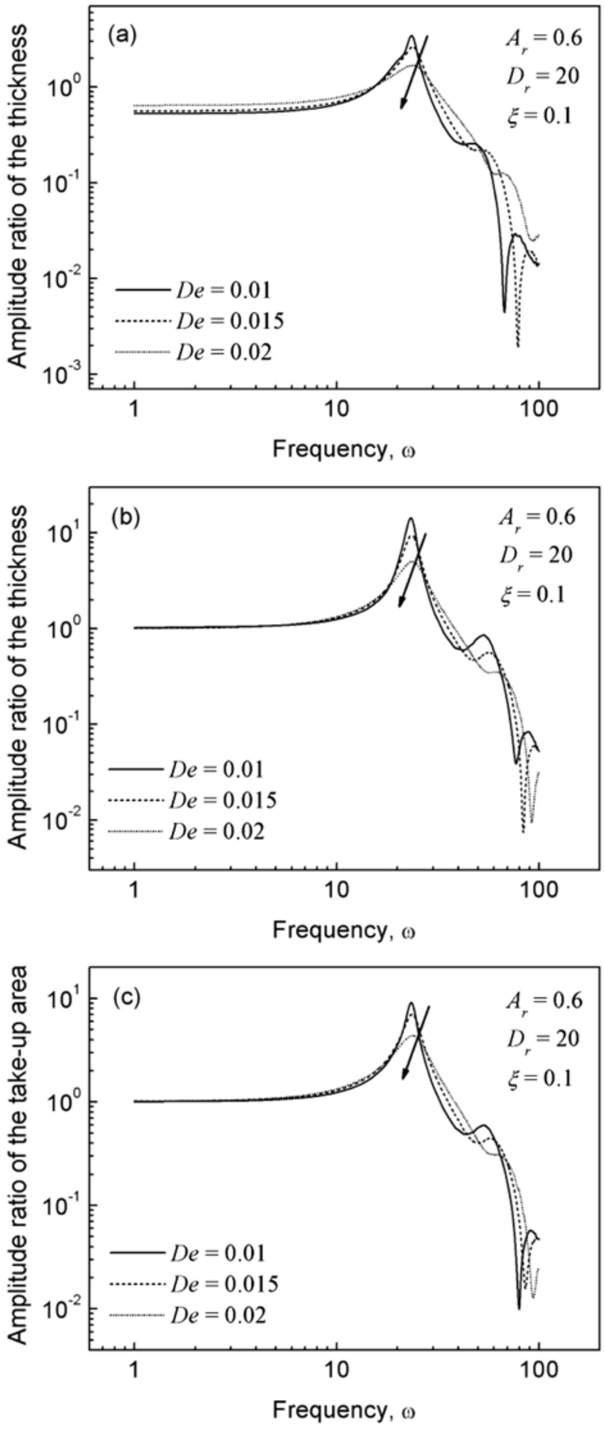

Fig. 8. Frequency response of (a) film thickness at edges, (b) film thickness at center, (c) take-up area for extension-thickening fluids with different Deborah numbers (ε=0.015, ξ=0.1).

other words, increasing fluid viscoelasticity (or Deborah number) makes the system more sensitive in the extension-thinning fluid case. As for extension-thickening fluids, on the other hand, as displayed in Fig. 8, the sensitivity result is the opposite, clearly showing less sensitive behavior with increasing De. It has been confirmed that the effect of fluid viscoelasticity on film casting sensitivity exhibits a dichotomous behavior, depending on whether the fluid is extension-thickening or extension-thinning, e.g., LDPE for extension-thickening or HDPE for extension-thinning with PTT model parameter ξ=0.1 or 0.7, respectively.

The trend of the abovementioned sensitivity results is closely linked with the stability ones, indicating the system is more sensitive to ongoing disturbances when it moves toward an unstable draw resonance state. For instance, the viscoelasticity of extension-thickening fluids, which can afford to stabilize the process system, makes the system less sensitive.

CONCLUSIONS

The sensitivity of the isothermal viscoelastic film casting process has been investigated via transient simulations for a two-dimensional (2-D) viscoelastic simulation model equipped with the Phan-Thien-Tanner (PTT) constitutive equation by incorporating a finite element method. The frequency response method using a 2-D model considered in this study gives more reasonable results than the 1-D case in that the 2-D model is capable of separately predicting the sensitivity of film thickness along with film width direction with respect to a sinusoidal disturbance. Various effects of process conditions on the sensitivity have been elucidated by introducing a sinusoidal perturbation at take-up speed. It has been found that the film thickness is more sensitive than the film width at take-up position, and also the sensitivity of film thickness at the center is more affected by a sinusoidal disturbance than that of the film thickness at the edges. As clarified in the stability theory, the fluid viscoelasticity can make the system more sensitive or less sensitive to any disturbances, depending on whether the fluid is extension-thickening or thinning.

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NOMENCLATURE

A_r : aspect ratio defined as the flow distance divided by the film width
$\mathbf{D}$: dimensionless strain rate tensor
De : Deborah number defined as the ratio of material time and processing time
D_r : drawdown ratio defined as the take-up speed divided by the extrusion speed
e : dimensionless film thickness
$\mathbf{n}$: normal vector
t : dimensionless time
$\mathbf{v}$: dimensionless velocity vector
w : dimensionless film width
δ : magnitude of disturbance
ε, ξ : PTT model parameters
η_0 : zero shear viscosity
λ : material relaxation time
ω : frequency
σ : dimensionless total stress tensor
τ : dimensionless extra stress tensor

Subscripts

0 : die-exit position
L : take-up position
x : machine direction
y : transverse direction

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