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Improved system identification method for Hammerstein-Wiener processes

Su Whan Sung*,†, Cheol Ho Je*, Jietae Lee*, and Dong Hyun Lee**

*Department of Chemical Engineering, Kyungpook National University, 1370 Sankyeok-dong, Buk-gu, Daegu 702-701, Korea
**Department of Chemical Engineering, Sungkyunkwan University, 300 Chunchun, Jangan, Suwon 440-746, Korea
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Abstract−We propose a new system identification method for Hammerstein-Wiener processes, in which an input static nonlinear block, a linear dynamic block, and an output static nonlinear block are connected in a series. The proposed method can estimate the model parameters in a very simple way without solving the full-dimensional nonlinear optimization problem by activating the process with a specially designed test signal, composed of a relay feedback signal, a binary signal and a multi-step signal. The proposed method analytically identifies the output nonlinear static function and the input nonlinear static function from the relay signal and the multi-step signal, respectively. The linear dynamic subsystem is identified from the relay feedback signal and the binary signal with existing well-established linear system identification methods. We demonstrate with a simple example that the proposed method can be successfully applied to identify the Hammerstein-Wiener-type nonlinear process.

Key words: Relay Feedback Method, Identification, Test Signal, Nonlinearity, Hammerstein-Wiener-type Nonlinear Process

INTRODUCTION

Block-oriented nonlinear models that consist of linear dynamic subsystem and memoryless nonlinear static functions such as Wiener, Hammerstein, and Hammerstein-Wiener models [1] have been often used to describe the nonlinear dynamics of many chemical, electrical, and biological processes. In this study, we will focus on the identification of the Hammerstein-Wiener nonlinear system in which an input nonlinear static function, a linear dynamic subsystem, and an output nonlinear static function are connected in a series. Several identification methods for Hammerstein-Wiener nonlinear processes have been proposed. An optimal two-stage identification algorithm [2] for Hammerstein-Wiener processes has been proposed. The approach uses singular value decomposition (SVD) to reduce the number of the model parameters after estimating oversized adjustable model parameters. Also, a blind approach [3] has been proposed, where internal variables are recovered from the oversampled process output and then the linear and nonlinear blocks are identified. Zhu [4] proposed an iterative relaxation algorithm to identify Hammerstein-Wiener nonlinear processes, and Crama and Schoukens [5] used a multi-sine signal to identify Hammerstein-Wiener nonlinear systems. A binary test signal [6] has been proposed to activate Hammerstein processes, which simplifies significantly the parameter estimation problem by separating the identification problem of the linear dynamic subsystem from that of the input nonlinear static function. Lee et al. [7] extended this to multi-input-multi-output (MIMO) Hammerstein-Wiener nonlinear processes. However, none of the previous approaches could separate the identification problem of the linear dynamic subsystem from that of the output nonlinear static function. Recently, a new identification method [8] has been proposed to separate the three identification problems of the input nonlinear static function, the linear dynamic subsystem, and the output nonlinear static function using three special test signals of two binary input signals with different sizes and a multi-step signal. From two binary input tests, it estimates the model parameters of the output nonlinear static function by solving a one-dimensional nonlinear optimization problem. After the output nonlinear static function is identified, the linear dynamic subsystem is identified by existing linear system identification methods [9-15], and the input nonlinear static function is analytically identified from the process data for the multi-step signal without any iterative optimization.

On the other hand, various relay feedback methods have been proposed and applied to various purposes since the original relay feedback method was proposed [16-26]. But, all the previous approaches cannot manipulate output nonlinearities and static disturbances simultaneously. Recently, a new relay feedback method [27] has been proposed in order to design an approximate sinusoidal test signal of the ultimate frequency of the process, which can be applied to nonlinear processes with output nonlinearities and static disturbances. The method has been successfully applied to identify Wiener processes with static disturbances.

In this research, we improve the previous strategy [8] to identify Hammerstein-Wiener processes by applying the recently proposed relay feedback method [27]. The proposed method activates the process using a relay feedback signal followed by a binary signal and a multi-step signal to separates the three identification problems of the input nonlinear static function, the linear dynamic subsystem, and the output nonlinear static function, which significantly simplifies the system identification procedure. It analytically identifies the output nonlinear function and the input nonlinear function from the activated process data sets by the relay signal and the multi-step signal, respectively, without solving any nonlinear optimization problems. And, the linear dynamic subsystem is identified from the relay feedback signal and the binary signal using existing well-defined linear system identification methods. We demonstrate with a simple example that the proposed method can be successfully applied to

†To whom correspondence should be addressed.
E-mail: suwhansung@knu.ac.kr

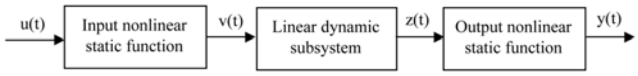

Fig. 1. Hammerstein-Wiener nonlinear process.

identify the Hammerstein-Wiener-type nonlinear process.

HAMMERSTEIN-WIENER NONLINEAR PROCESSES

In this research, a single-input-single-output Hammerstein-Wiener nonlinear process is considered, as shown in Fig. 1. It can be represented by

$$v(t)=f(u(t))=p_1u(t)+p_2u^2(t)+\cdots+p_mu^m(t) \tag{1}$$

$$\frac{d\mathbf{x}(t)}{dt}=\mathbf{A}\mathbf{x}(t)+\mathbf{B}v(t-T_d) \tag{2}$$

$$z(t)=\mathbf{C}\mathbf{x}(t)$$

$$z(t)=g^{-1}(y(t))=q_1y(t)+q_2y^2(t)+\Lambda+q_ry^r(t) \tag{3}$$

Here, u(t) and y(t) denote the process input and the process output, respectively. (1), (2) and (3) represent the input nonlinear static function, the linear dynamic subsystem and the output nonlinear static function, respectively. The variable $\mathbf{x}(t)$ is the n-dimensional state, and the intermediate variables v(t) and z(t) are the output of the input nonlinear static function and the output of the linear dynamic subsystem, respectively. T_d denotes the input time delay. The system matrices of **A**, **B**, and **C** can have the following forms.

$$\mathbf{A}=\begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & -a_n \\ 1 & 0 & 0 & \cdots & 0 & -a_{n-1} \\ 0 & 1 & 0 & \cdots & 0 & -a_{n-2} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & -a_2 \\ 0 & 0 & 0 & \cdots & 1 & -a_1 \end{bmatrix} \tag{4}$$

$$\mathbf{B}=[b_n\ b_{n-1}\ b_{n-2}\ \cdots\ b_2\ b_1]^T \tag{5}$$

$$\mathbf{C}=[0\ 0\ 0\ \cdots\ 0\ 1] \tag{6}$$

The main issue of this research is to estimate the system matrices of **A**, **B**, T_d, p_i, i=1, 2, ⋯, m, q_i, i=1, 2, ⋯, r from the sampled output data y(t) and the given input data u(t).

Remarks

1. The proposed identification procedure can be straightforwardly applied to other linear model representations such as discrete time, continuous time, state space, and time series instead of the continuous time state space representation of (2).

2. We assume that the initial state is zero. Then, the linear dynamic subsystem of (2) is equivalent to the following transfer function model.

$$\frac{z(s)}{v(s)}=\frac{b_1s^{n-1}+b_2s^{n-2}+b_{n-1}s+b_n}{s^n+a_1s^{n-1}+\cdots+a_{n-1}s+a_n}\exp(-T_ds) \tag{7}$$

3. As in (1) and (3), the polynomial form is used for the input and output nonlinear static functions. If the polynomial form is inadequate, any other static function forms which are linear with respective to the adjustable parameters such as spline functions, piecewise linear functions, and orthogonal series can be used.

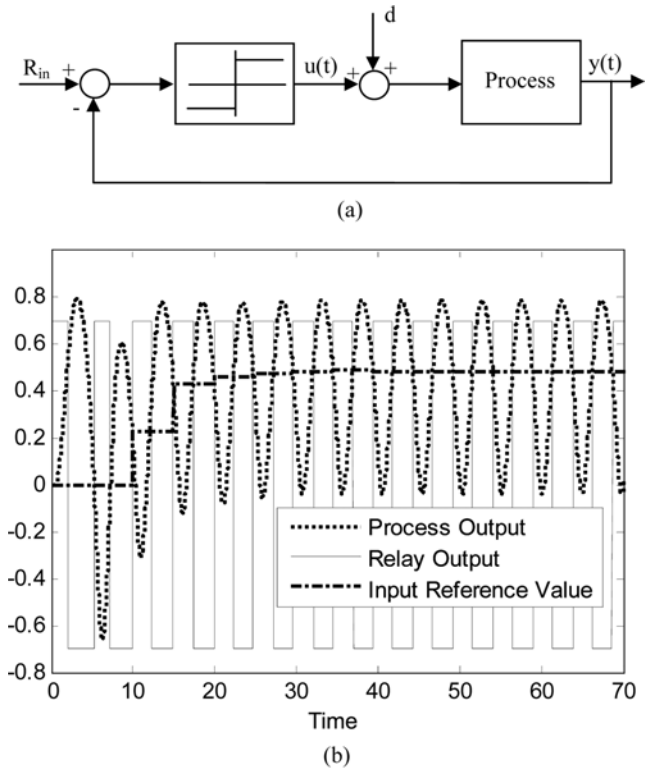

Fig. 2. Schematic diagram of the relay feedback method (a); typical dynamic behaviors of the relay feedback method for a static disturbance (b).

RELAY FEEDBACK METHOD

Since we use the previously proposed relay feedback method [27] in the proposed identification strategy, we need to introduce it briefly. Fig. 2(a) shows the schematic diagram for the relay feedback method [27]. u(t) and y(t) are the relay output and the process output, respectively. d represents a static disturbance. R_{in} is the input reference value of the relay, respectively. Fig. 2(b) shows typical behaviors of the relay feedback method. Here, the input static disturbance (d) enters at time=0.

The relay method [27] designs the symmetric relay output of the ultimate frequency through the following procedure. Step 1, obtain one-period-oscillation in the same way as the ordinary relay feedback method [16]. Step 2, enter the lower value of the relay and wait for a time equal to the half of the previous period. This enforces the symmetry of the relay output in the cyclic steady state of one frequency. And, after the time equal to the half of the previous period, set R_{in}=y. This forces the two crossing points in one period between the process output and the input reference value to converge to the same value. Step 3, enter the upper value of the relay and wait for the process output to cross the input reference value (R_{in}) for the relay. Repeat step 2 and step 3 until a cyclic steady state is obtained.

We should focus on two remarkable aspects for the relay feedback method. One is that it always guarantees a symmetric relay output when a cyclic steady state of one period is obtained because it sets exactly the time length of the lower value of the relay to the half-period. The other remarkable aspect is that the relay method updates R_{in} in order to reject the effects of the static disturbance. These two aspects allow us to generate an approximate sinusoidal signal (equivalently, a symmetric relay output) of the ultimate frequency under circumstances of static disturbances and output nonlinearities. For example, it provides a symmetric relay signal of the ultimate period for Wiener processes with static disturbances. Also, it can be applied to the Hammerstein-Wiener nonlinear process of Fig. 1 because the binary relay signal becomes a shifted binary signal after the input nonlinear static function, which can be considered as the case of the Wiener process with static disturbances.

PROCESS ACTIVATION

The proposed identification method activates the Hammerstein-Wiener process with the relay feedback signal followed by a binary signal and a multi-step signal. For example, consider the test signal of Fig. 3. The process is initially activated by the relay feedback method [27] from t=0 to t=40 followed by a binary signal from t=40 to t=80 and a multi-step signal from t=80 to t=120. The upper value of the binary signal should be the same as the upper value of the relay and the lower value of the binary signal should be zero. The binary signal and the multi-step signal of Fig. 3 is a random binary signal of 0 and 1 and a uniformly distributed random noise between −0.2 and 1.0, respectively, with the minimum switching time=1, satisfying the following overriding conditions. If the process output is bigger than $y_{max}-\alpha$, the minimum value of the signal enters in an overriding way. The maximum value enters if the process output is smaller than $y_{min}+\alpha$. Here, y_{max} and y_{min} are the peak value and the valley value in the cyclic steady state of the process output in the relay feedback signal part. α is a small value. The overriding conditions are to force the activated process output by the binary and the multi-step signal to stay in the output range of the cyclic steady state part in the relay feedback signal part (the reason will be explained later).

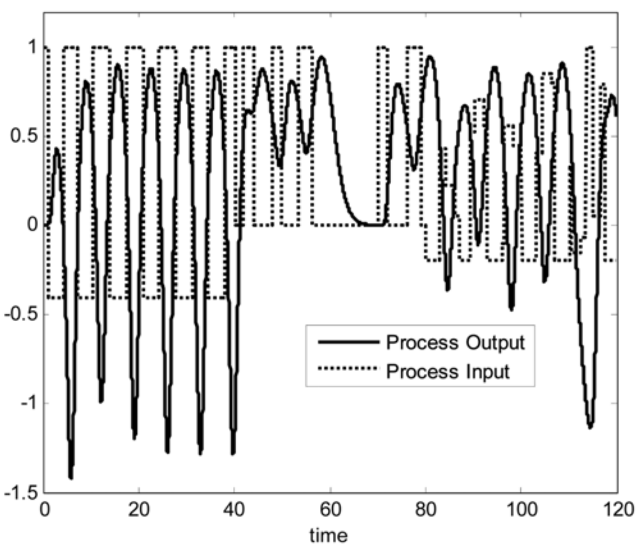

Fig. 3. Typical responses of the process for the proposed activation signal.

Remarks

1. The proposed activation method requires only one experiment, while the previous method [8] using two binary signals and one multi-step signal requires two experiments.

2. We can determine appropriately the scale of the minimum switching time for the binary signal and the multi-step signal on the basis of the period of the relay feedback signal. Meanwhile, the previous approach [8] has no a priori information to set the minimum switching time.

3. Theoretically, we do not need the binary signal in the proposed test signal because the relay feedback signal includes various frequency components if we choose different magnitudes for the upper value and the lower value of the relay. But, we recommend the usage of the binary signal from a practical point of view because the ultimate frequency quantity of the relay feedback signal can be too big compared to the other frequency quantities, resulting in unbalanced system identification results.

PROPOSED IDENTIFICATION METHOD

The procedure of the proposed identification method is as follows. First, the proposed method analytically identifies the output nonlinear static function of the Hammerstein-Wiener process from the relay feedback signal without solving any iterative nonlinear optimization problems. Second, the linear dynamic subsystem is obtained with existing well-developed linear system identification methods from the relay signal and the binary signal. Third, the input nonlinear static function is analytically identified from the whole test signal.

1. Identification of the Output Nonlinear Static Function

We use the cyclic steady state part of the relay feedback signal of Fig. 3 to estimate the output nonlinear static function. Assume that u_u and u_l are the upper value and the lower value of the relay. The corresponding upper value and the lower value of the output of the input nonlinear static function will be $v_u=f(u_u)$ and $v_l=f(u_l)$. Then, the first two terms in the Fourier series of the output of the input nonlinear static function are

$$v(t)\approx\frac{v_u+v_l}{2}-\frac{4[(v_u-v_l)/2]}{\pi}\sin(\omega t) \tag{8}$$

The higher-order harmonic terms of the symmetric output are relatively small compared with the first two terms. Also, the process dynamics attenuates much more the higher-order harmonic terms than the first two terms. So, the signal of z(t) in cyclic steady state can be approximated by the following sinusoidal signal.

$$z(t)\approx\frac{[v_u+v_l]G(0)}{2}-\frac{4[(v_u-v_l)/2]|G(j\omega)|}{\pi}\sin(\omega t) \tag{9}$$

where G(0) and G(jω) are the frequency responses of the linear dynamic subsystem at zero and the relay (ultimate) frequency, respectively. From (3) and (9),

$$\frac{z(t)}{M_2}=\frac{q_1}{M_2}y+\frac{q_2}{M_2}y^2+\frac{q_3}{M_2}y^3+\cdots+\frac{q_r}{M_2}y^r\approx\frac{M_1}{M_2}-\sin(\omega t) \tag{10}$$

where $M_1=\frac{[v_u+v_l]G(0)}{2}$, $M_2=\frac{4[(v_u-v_l)/2]|G(j\omega)|}{\pi}$.

Therefore, we can estimate the normalized output nonlinear static function by solving the following optimization problem of (11), which can be solved analytically by the least square method.

$$\min_{\hat{q}} \sum_{i=0}^{N} (\sin(\omega t_i) + \hat{q}_0 + \hat{q}_1 y_i + \hat{q}_2 y_i^2 + \cdots + \hat{q}_r y_i^r)^2 \tag{11}$$

where $\hat{q}_i$, i=1, 2, 3, ⋯ is an estimate corresponding to q_i/M_2, i=1, 2, 3, ⋯ and $\hat{q}_0$ corresponds to $-M_1/M_2$. And, $\bar{z}(t)=\hat{q}_1 y(t)+\hat{q}_2 y^2(t)+\cdots+\hat{q}_r y^r(t)$ is the estimated model output of the output nonlinear static function, corresponding to $z(t)/M_2$. N is the number of the measurements for one period in the cyclic steady state.

Remarks

1. We can determine the order of the polynomials more systematically by minimizing a joint cost function (for example, such as Akaike's information theoretic criterion), which includes the modeling error term and a penalty term for model complexity [9].

2. The previous approach [8] should solve an one-dimensional nonlinear optimization method to obtain the output nonlinear static function. Meanwhile, the proposed approach estimates the output nonlinear static function analytically.

2. Identification of the Linear Dynamic Subsystem

Once the output nonlinear static function ($\hat{q}_i$, i=2, ⋯ r) is identified from the relay feedback signal, we can calculate the output of the linear dynamic subsystem with $\bar{z}(t)=\hat{q}_1 y(t)+\hat{q}_2 y^2(t)+\cdots+\hat{q}_r y^r(t)$ for the relay signal part and the binary signal part. To guarantee acceptable data accuracy of $\bar{z}(t)$ in the binary signal region, the process output of the binary signal should stay in the range of the cyclic steady state of the relay feedback signal part because the model parameters of $\hat{q}_i$, i=2, ⋯ r are estimated on the basis of the cyclic steady state data. This is why we need the overriding conditions in section 4.

The binary input of the linear dynamic subsystem can be assumed to be v(t)=u(t) for the upper value and the lower (zero) value of the binary signal and $v(t)=\hat{b}$ for the lower value of the relay signal without loss of generality because we have a degree of freedom to change **B** without changing the overall input-output dynamics. Here, $\hat{b}$ should be estimated. Now, we can identify the linear dynamic subsystem from $v(t_i)$ and the estimated output ($\bar{z}(t_i)$) of the linear dynamic subsystem by applying existing well-established linear system identification methods such as prediction error, instrumental variable methods [9-11], and subspace system identification methods [12-15]. In this research, we use the following prediction error identification method [10]:

$$\min_{\hat{A},\hat{B},\hat{T}_d,\hat{b}} \left[V(\hat{A},\hat{B},\hat{T}_d,\hat{b}) = \frac{0.5}{N_{R-B}} \sum_{i=1}^{N_{R-B}} (\bar{z}(t_i)-\hat{z}(t_i))^2 \right] \tag{12}$$

subject to

v(t)=u(t) for the upper value and the lower value of the binary signal (13)

$v(t)=\hat{b}$ for the lower value of the relay signal (14)

$$\frac{d\hat{\mathbf{x}}(t)}{dt} = \hat{\mathbf{A}}\mathbf{x}(t) + \hat{\mathbf{B}}\hat{v}(t-\hat{T}_d) \tag{15}$$

$$\hat{z}(t) = \mathbf{C}\hat{\mathbf{x}}(t) \tag{16}$$

$$\bar{z}(t) = \hat{q}_1 y(t) + \hat{q}_2 y^2(t) + \cdots + \hat{q}_r y^r(t) \tag{17}$$

where $\bar{z}(t)$, $\hat{z}(t)$ and y(t) denote the estimated output of the linear dynamic subsystem, the predicted model output, and the measured process output, respectively. N_{R-B} is the number of the measured process outputs in the relay signal part and the binary signal part, belonging to the output range of the cyclic steady state in the relay signal part.

3. Identification of the Input Nonlinear Static Function

Once the output nonlinear static function and the linear dynamic subsystem are identified, we can identify the input nonlinear static function by minimizing the following cost function from the data sets of the whole test signal in Fig. 3 and the corresponding estimated output of the linear dynamic subsystem.

$$\min_{\hat{p}_i, j=1,2,\Lambda,m} \left[V(\hat{p}_1,\hat{p}_2,\Lambda,\hat{p}_m) = \frac{0.5}{N_{R-B-M}} \sum_{i=1}^{N_{R-B-M}} (\bar{z}(t_i)-\hat{z}(t_i))^2 \right] \tag{18}$$

subject to

$$\hat{v}(t) = \hat{p}_1 u(t) + \hat{p}_2 u^2(t) + \cdots + \hat{p}_m u^m(t) \tag{19}$$

$$\frac{d\hat{x}(t)}{dt} = \hat{A}\hat{x}(t) + \hat{B}\hat{v}(t-\hat{T}_d) \tag{20}$$

$$\hat{z}(t) = C\hat{x}(t) \tag{21}$$

$$\bar{z}(t) = \hat{q}_1 y(t) + \hat{q}_2 y^2(t) + \cdots + \hat{q}_r y^r(t) \tag{22}$$

Here $\bar{z}(t)$, $\hat{z}(t)$ and y(t) denote the estimated output of the linear dynamic subsystem, the predicted model output, and the measured process output corresponding to the whole test signal, respectively. N_{R-B-M} is the number of data points corresponding to the whole test signal of Fig. 3, belonging to the output range of the cyclic steady state in the relay signal part. $\hat{A}$, $\hat{B}$, and $\hat{T}_d$ represent the estimates of the linear dynamic subsystem obtained in the previous section.

The derivatives of the cost function with respect to the adjustable parameters $\hat{p}_i$, i=1, 2, ⋯, m can be obtained as follows. From (18), we have

$$\frac{\partial V(\hat{P})}{\partial \hat{P}} = -\frac{1}{N_{R-B-M}} \sum_{i=1}^{N_{R-B-M}} (\bar{z}(t_i)-\hat{z}(t_i)) \frac{\partial \hat{z}(t_i)}{\partial \hat{P}} \tag{23}$$

$$\frac{\partial \hat{z}(t)}{\partial \hat{P}} = \left[\frac{\partial \hat{z}(t)}{\partial \hat{p}_1} \frac{\partial \hat{z}(t)}{\partial \hat{p}_2} \cdots \frac{\partial \hat{z}(t)}{\partial \hat{p}_m} \right] \tag{24}$$

where $\hat{P} = [\,\hat{p}_1\, \hat{p}_2 \cdots \hat{p}_m\,]^T$.

From (19)-(21), we have

$$\frac{\partial \hat{v}(t)}{\partial \hat{p}_i} = u^i(t),\ i=1, 2, \ldots, m \tag{25}$$

$$\frac{d}{dt}\left(\frac{\partial \hat{x}(t)}{\partial \hat{p}_i}\right) = \hat{A}\frac{\partial \hat{x}(t)}{\partial \hat{p}_i} + \hat{B}\frac{\partial \hat{v}(t-\hat{T}_d)}{\partial \hat{p}_i} \tag{26}$$

$$\frac{\partial \hat{z}(t)}{\partial \hat{p}_i} = C\frac{\partial \hat{x}(t)}{\partial \hat{p}_i} \tag{27}$$

In the differential Eq. (26), the initial values of $\partial\hat{x}(t)/\partial\hat{p}_i|_{t=0}$, i=1, 2, …, m are zero because $\hat{p}_i$ cannot change the initial values of the state vector. Then, we can solve the differential Eqs. (25)-(27) and calculate the first derivative of the cost function of (23).

The second derivative of the cost function is

$$\frac{\partial^2 V(\hat{P})}{\partial \hat{P}^2} = -\frac{1}{N_{R-B-M}} \sum_{i=1}^{N_{R-B-M}} (\bar{z}(t_i)-\hat{z}(t_i)) \frac{\partial^2 \hat{z}(t_i)}{\partial \hat{P}^2} + \frac{1}{N_{R-B-M}} \sum_{i=1}^{N_{R-B-M}} \left[\frac{\partial \hat{z}(t_i)}{\partial \hat{P}}\right] \left[\frac{\partial \hat{z}(t_i)}{\partial \hat{P}}\right]^T \tag{28}$$

From (25), it is clear that $\partial^2 v(t)/\partial\hat{p}_i\partial\hat{p}_j$, i, j=1, 2, …, m are zero. The second derivative of the state is also zero at t=0 (i.e., $\partial^2\hat{x}(t)/\partial\hat{p}_i\partial\hat{p}_j|_{t=0}=0$, i, j=1, 2, …, m) because the initial values of $\hat{x}(t)$ are always constants. Then, $\partial^2\hat{x}(t)/\partial\hat{p}_i\partial\hat{p}_j$ and $\partial^2\hat{z}(t)/\partial\hat{P}^2$ are always zero. Therefore, the following equations are obtained from (28):

$$\frac{\partial^2 V(\hat{P})}{\partial\hat{P}^2}=\frac{1}{N_{R-B-M}}\sum_{i=1}^{N_{R-B-M}}\left[\frac{\partial\hat{z}(t_i)}{\partial\hat{P}}\right]\left[\frac{\partial\hat{z}(t_i)}{\partial\hat{P}}\right]^T \tag{29}$$

$$\frac{\partial^j V(\hat{P})}{\partial\hat{P}^j}=0,\ j>2 \tag{30}$$

This means that the cost function of (18) is quadratic. And, we can obtain the following linear equation.

$$\frac{\partial V(\hat{P})}{\partial\hat{P}}=\left.\frac{\partial V(\hat{P})}{\partial\hat{P}}\right|_{\hat{P}=0}+\left.\frac{\partial^2 V(\hat{P})}{\partial\hat{P}^2}\right|_{\hat{P}=0}\hat{P} \tag{31}$$

Therefore, without any iterative procedure, the optimization problem of (18) can be analytically solved by setting $\partial V(\hat{P})/\partial\hat{P}=0$ in (31). The optimal solution is

$$\hat{P}^*=-\left[\left.\frac{\partial^2 V(\hat{P})}{\partial\hat{P}^2}\right|_{\hat{P}=0}\right]^{-1}\left[\left.\frac{\partial V(\hat{P})}{\partial\hat{P}}\right|_{\hat{P}=0}\right] \tag{32}$$

In summary, we can analytically obtain the optimal solution of (32) for the input nonlinear static function model from (23)-(27), (29) and $\hat{z}(t)=0$.

CASE STUDY

Consider the following Hammerstein-Wiener nonlinear process in which the linear dynamic subsystem is a third order plus time delay model and the input and the output nonlinear function are exponential functions.

$$v(t)=1-\exp(-3u(t))\ \text{for}\ u(t)\geq 0 \tag{33}$$

$$v(t)=-1+\exp(3u(t))\ \text{for}\ u(t)<0 \tag{34}$$

$$\frac{z(s)}{v(s)}=\frac{\exp(-s)}{s^3+3s^2+3s+1} \tag{35}$$

$$y(t)=0.2z(t)+(1-\exp(2.5z(t)) \tag{36}$$

Fig. 3 shows the activated process output. We solved the optimization problem of (11) with the input-output data sets of one period in the cyclic steady state for the relay feedback signal and obtained the following inverse model of the output nonlinear static function.

$$\hat{z}(t)=0.9160\hat{y}(t)+0.4082\hat{y}^2(t)+0.2189\hat{y}^3(t)+0.2725\hat{y}^4(t)+0.0458\hat{y}^5(t)-0.1955\hat{y}^6(t)-0.0991\hat{y}^7(t) \tag{37}$$

Fig. 4 shows that the rescaled model of (37) is close to the actual one of (36). We used the continuous-time prediction error identification method [10] which estimates the model parameters of the linear dynamic subsystem minimizing the prediction error of (12). We obtained the following second order plus time delay model for the linear dynamic subsystem from the data sets of the relay signal and the binary signal and the calculated $\bar{z}(t)$.

$$\frac{\hat{z}(s)}{\hat{v}(s)}=\frac{0.9439\exp(-1.25s)}{s^2+1.0485s+0.4089} \tag{38}$$

Fig. 5 compares the rescaled model of (38) with the actual linear dynamic subsystem of the Hammerstein-Wiener process. Even though

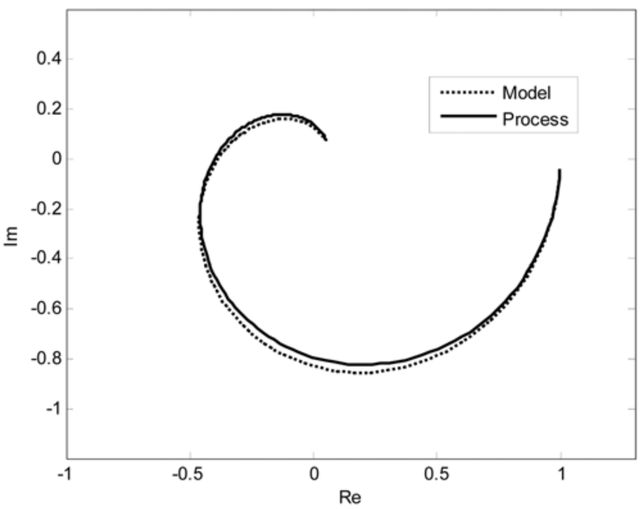

Fig. 5. Nyquist plot of the estimated linear dynamic subsystem.

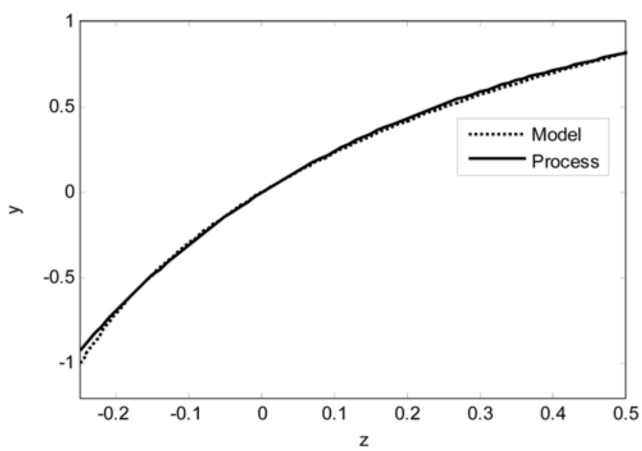

Fig. 4. Estimated output nonlinear static function.

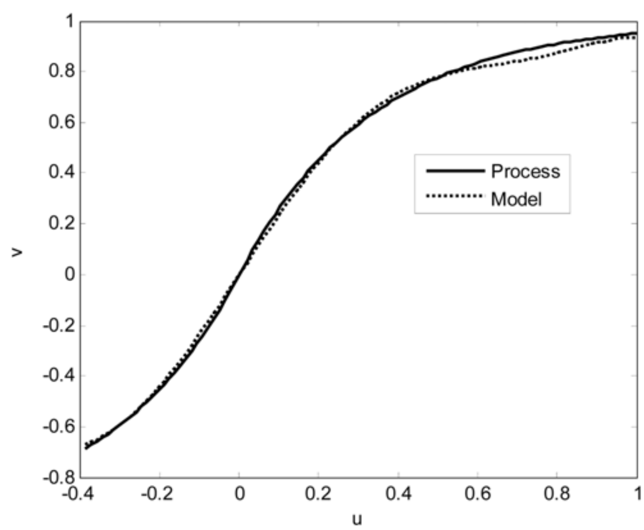

Fig. 6. Estimated input nonlinear static functions.

there are plant/model mismatches as shown in (35) and (38), the identification results are acceptable. Next, the input nonlinear static function is estimated by the analytical solution of (32) from the whole data sets of the test signal and the measured process output. The following fifth order polynomial model for the input nonlinear static function is analytically obtained.

$$\hat{v}(t)=2.545u(t)+0.046u^2(t)-5.647u^3(t)+1.399u^4(t)+7.322u^5(t)-4.692u^6(t) \quad (39)$$

We realize that the proposed identification method is acceptable as shown in Fig. 6.

CONCLUSIONS

A new system identification method has been proposed to obtain the Hammerstein-Wiener model without solving full-dimensional nonlinear optimization problems. It activates the process with one experiment using a specially designed test signal composed of a relay feedback signal, a binary signal and a multi-step signal to significantly simplify the identification procedure. The proposed method identifies the output nonlinear function and the input nonlinear function analytically without solving any iterative optimization problems. The linear dynamic subsystem is identified by existing well-established linear system identification methods. We demonstrate with a simple example that the proposed method can be successfully applied to identify the Hammerstein-Wiener-type nonlinear process.

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