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Adaptive monitoring statistics with state space model updating based on canonical variate analysis

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Abstract−A new multivariate statistical model updating by using a recursive state space model updating based on CVA is proposed. The CVA-based monitoring techniques have been researched to detect and isolate process abnormalities in dynamic processes. Two monitoring indices are defined for fault detection, and a state space model updating procedure is developed by using mean, covariance, and correlation updating based on forgetting factor as well as the recursive Cholesky factor updating. To adjust forgetting factors according to variation of process state, the forgetting factor updating criteria are introduced. The proposed method is applied to benchmark models of a continuous stirred tank reactor with a first order reaction and the Tennessee Eastman process (TEP) under transient and time-varying operating conditions. Through the simulation results, we expect that the proposed approach can be applied to time-varying and dynamic processes under transient state.

Key words: Adaptive Monitoring, State Space Model, Canonical Variate Analysis

INTRODUCTION

A large number of multivariate statistical process control (MSPC) techniques such as principal component analysis (PCA) and partial least squares (PLS) based statistics have been developed to detect and isolate process abnormalities in modern chemical processes with high dimensional-correlated variables. Though these approaches show good fault detection and identification performance under stationary operating conditions, a real process is often faced with wanted or unwanted changes of process operating conditions. Thus, it is another important issue to develop recursive methods for adaptation to a time-varying or transient process.

Canonical variate analysis (CVA) has been adopted to monitor state of operating condition in a dynamic process [1]. Several researchers [2,3] show that a CVA-based approach is more powerful than the other conventional dynamic approaches to detect and identify process abnormalities. The paper focuses on a state space model updating with CVA.

It is organized as follows. In section 2, CVA-based state space model and modeling procedure are introduced. A general state space model derived from vector auto-regressive moving average (VARMA) process [4] is employed. In section 3, CVA-based static process monitoring techniques are discussed. In section 4, we discuss updating methods for mean vector, covariance matrix, the scaled Hankel matrix, forgetting factors, state estimation matrix, and noise extraction matrix. And then confidence level updating of the proposed monitoring statistics is given in section 5. Scenario-based application results are shown and explained in section 6.

STATE SPACE MODELING USING CVA

A dynamic process monitoring technique which is based on state space modeling with CVA has been proposed by Negiz and Cinar [1]. The state space model derived from VARMA is given as follows:

$$\mathbf{x}_{t+1}=\mathbf{A}\mathbf{x}_t+\mathbf{w}_t,\ \mathbf{w}_t=\mathbf{B}\mathbf{v}_t$$
$$\mathbf{y}_t=\mathbf{C}\mathbf{x}_t+\mathbf{v}_t \tag{1}$$

where $\mathbf{x}_t\in\Re^k$ is state vector, $\mathbf{y}_t\in\Re^p$ observation vector, $\mathbf{w}_t$ stochastic disturbance, $\mathbf{v}_t$ measurement noise, $\mathbf{A}$ state matrix, and $\mathbf{C}$ output matrix, respectively. The sub-matrices, $\mathbf{A}$ and $\mathbf{C}$ in Eq. (1) can be estimated with least squares method [4,5].

$$\begin{bmatrix}\mathbf{A}\\ \mathbf{C}\end{bmatrix}=\mathrm{E}\begin{bmatrix}\mathbf{x}_{t+1}\mathbf{x}_t^T\\ \mathbf{y}_t\mathbf{x}_t^T\end{bmatrix}\mathrm{E}(\mathbf{x}_t\mathbf{x}_t^T)^{-1} \tag{2}$$

To identify the state space model by using the solution of least squares in Eq. (2), the state vector, $\mathbf{x}_t$, is first assumed. Akaike [6] proposed canonical variables as a candidate of the state vector.

The estimator of canonical variables can be formulated from results of the generalized singular value decomposition of the normalized Hankel matrix in Eq. (1). A Hankel matrix using time series data is calculated from matrix computation of past and future stacked data. To explain the calculation procedure of canonical variable estimator, the following three stacked vectors are defined.

$$\mathbf{p}_t=[\mathbf{y}_{t-1}^T\ \cdots\ \mathbf{y}_{t-1}^T]^T \tag{3}$$

$$\mathbf{f}_t=[\mathbf{y}_{t+l-1}^T\ \cdots\ \mathbf{y}_t^T]^T \tag{4}$$

$$\overleftarrow{\mathbf{p}}_t=[\mathbf{y}_t^T\ \cdots\ \mathbf{y}_{t-l}^T]^T \tag{5}$$

where $\mathbf{p}_t$, $\mathbf{f}_t$, and $\overleftarrow{\mathbf{p}}_t$ denote mean centered and scaled past, future, and expanded past stacked vector of observations and l is the number of the lag or the lead. The normalized past and future stacked vectors can be estimated as

$$\mathbf{d}_{p,t}=\Sigma_{pp}^{-1/2}\mathbf{p}_t,\ \ \mathbf{d}_{f,t}=\Sigma_{ff}^{-1/2}\mathbf{f}_t \tag{6}$$

where $\Sigma_{pp}=\mathrm{E}(\mathbf{p}_t\mathbf{p}_t^T)$ and $\Sigma_{ff}=\mathrm{E}(\mathbf{f}_t\mathbf{f}_t^T)$ (E(•) denotes the expectation

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operator) [4]. The normalized past and future stacked vectors, $\mathbf{d}_{p,t}$ and $\mathbf{d}_{f,t}$ are defined as the scaled stacked future and past vectors at time t, respectively. The expectation of $\mathbf{d}_{f,t}$ under the condition of $\mathbf{d}_{p,t}$ can be expressed as

$$\hat{\mathbf{d}}_{f,t}=\Sigma_{ff}^{-1/2}\Sigma_{fp}\Sigma_{pp}^{-1/2}\mathbf{d}_{p,t} \tag{7}$$

where $\hat{\mathbf{d}}_{f,t}$ denotes the conditional expectation of $\mathbf{d}_{f,t}$ and Σ_{fp}, defined as $E(\mathbf{f}_t\mathbf{p}_t^T)$, is a well-known Hankel matrix. Thus, $\Sigma_{ff}^{-1/2}\Sigma_{fp}\Sigma_{pp}^{-1/2}$ indicates the scaled Hankel matrix. The scaled Hankel matrix can be factorized by singular value decomposition (SVD),

$$\Sigma_{ff}^{-1/2}\Sigma_{fp}\Sigma_{pp}^{-1/2}=\mathbf{U}^{\Sigma}\mathbf{S}^{\Sigma}\mathbf{V}^{\Sigma^T}\approx\mathbf{U}_k^{\Sigma}\mathbf{S}_k^{\Sigma}\mathbf{V}_k^{\Sigma^T} \tag{8}$$

where k represents the state order. $\mathbf{U}_k^{\Sigma}$ and $\mathbf{V}_k^{\Sigma}$ consist of the first k column vectors of $\mathbf{U}$ and $\mathbf{V}$, respectively, and the diagonal matrix $\mathbf{S}_k$ is the k×k principal submatrix of $\mathbf{S}$. Then, the past and future canonical variables at time t are given by

$$\mathbf{z}_t=\mathbf{U}_k^{\Sigma^T}\mathbf{d}_{f,t}=\mathbf{U}_k^{\Sigma^T}\Sigma_{ff}^{-1/2}\mathbf{f}_t=\mathbf{L}_k\mathbf{f}_t$$
$$\mathbf{m}_t=\mathbf{V}_k^{\Sigma^T}\mathbf{d}_{p,t}=\mathbf{V}_k^{\Sigma^T}\Sigma_{pp}^{-1/2}\mathbf{p}_t=\mathbf{J}_k\mathbf{p}_t \tag{9}$$

where $\mathbf{z}_t$ and $\mathbf{m}_t$ denotes the future and past canonical variables. These canonical variables satisfy

$$\mathbf{z}_t=\mathbf{S}_k^{\Sigma}\mathbf{m}_t \tag{10}$$

As an alternative approach of CVA [7], the Cholesky factor instead of the covariance matrix can be used in Eq. (8).

$$\mathbf{R}_{ff}^{-T}\Sigma_{fp}\mathbf{R}_{pp}^{-1}\approx\mathbf{U}_k^{R}\mathbf{S}_k^{R}\mathbf{V}_k^{R^T} \tag{11}$$

where $\mathbf{R}_{ff}$ and $\mathbf{R}_{pp}$ denote the Cholesky factor of Σ_{ff} and Σ_{pp}, respectively. Regardless of the choice of the covariance matrix and the Cholesky factor, the past canonical variables approximate to the state vector.

$$\mathbf{x}_t\approx\mathbf{m}_t=\mathbf{J}_k\mathbf{p}_t=\mathbf{V}_k^{\Sigma^T}\Sigma_{pp}^{-0.5}\mathbf{p}_t=\mathbf{V}_k^{R^T}\mathbf{R}_{pp}^{-T}\mathbf{p}_t \tag{12}$$

where $\mathbf{J}_k\equiv\mathbf{V}_k^{\Sigma^T}\Sigma_{pp}^{-0.5}=\mathbf{V}_k^{R^T}\mathbf{R}_{pp}^{-T}$ is referred to the state estimation matrix. Using the past canonical variables, Eq. (2) can be approximately replaced as follows:

$$\begin{bmatrix}\mathbf{A}\\ \mathbf{C}\end{bmatrix}=E\begin{bmatrix}\mathbf{m}_{t+1}\mathbf{m}_t^T\\ \mathbf{y}_t\mathbf{m}_t^T\end{bmatrix}E(\mathbf{m}_t\mathbf{m}_t^T)^{-1} \tag{13}$$

Thus, the state space model can be obtained from collected normal process data using Eqs. (9) and (13).

STATE SPACE MODEL BASED MONITORING

After state space model identification, monitoring statistics have to be defined to detect process abnormalities. Various researches [1-3,8] have defined a large number of monitoring statistics. In this paper, two monitoring indices [8] are proposed with state space model. One is a state monitoring index and the a other noise process monitoring index. The state vector of on-line observations can be calculated by using Eq. (12). A noise process vector can be extracted from the following relations.

$$\mathbf{w}_t=\mathbf{x}_{t+1}-\mathbf{A}\mathbf{x}_t$$
$$\mathbf{v}_t=\mathbf{y}_t-\mathbf{C}\mathbf{x}_t \tag{14}$$

Using canonical variables, the state vectors at t and t+1 can be approximated as

$$\mathbf{x}_{t+1}\approx\mathbf{m}_{t+1}=\mathbf{J}_k\mathbf{p}_{t+1}=[\mathbf{J}_k\ \ \mathbf{0}]\overleftarrow{\mathbf{p}}_t$$
$$\mathbf{x}_t\approx\mathbf{m}_t=\mathbf{J}_k\mathbf{p}_t=[\mathbf{0}\ \ \mathbf{J}_k]\overleftarrow{\mathbf{p}}_t \tag{15}$$

where $\mathbf{0}\in\Re^{k\times p}$ with all zero entries. Using Eqs. (14) and (15), the noise process vector can be defined as

$$\mathbf{e}_t=\begin{bmatrix}\mathbf{w}_t\\ \mathbf{v}_t\end{bmatrix}=\begin{bmatrix}[\mathbf{J}_k\ \ \mathbf{0}]-[\mathbf{0}\ \ \mathbf{A}\mathbf{J}_k]\\ [\mathbf{I}\ \ \mathbf{C}\mathbf{J}_k]\end{bmatrix}\overleftarrow{\mathbf{p}}_t=\Xi\overleftarrow{\mathbf{p}}_t \tag{16}$$

where $\Xi\equiv\begin{bmatrix}[\mathbf{J}_k\ \ \mathbf{0}]-[\mathbf{0}\ \ \mathbf{A}\mathbf{J}_k]\\ [\mathbf{I}\ \ \mathbf{C}\mathbf{J}_k]\end{bmatrix}$ is referred to as the noise extraction matrix.

By considering Mahalanobis distances of the state and noise vector, the monitoring statistics of them can be defined as

$$T_s^2=\mathbf{x}_t^T\mathbf{x}_t=\mathbf{p}_t^T\mathbf{J}_k^T\mathbf{J}_k\mathbf{p}_t\sim\frac{k(n^2-1)}{n(n-k)}F_\alpha(k,n-k)$$
$$T_{wv}^2=\mathbf{e}_t^T\mathbf{S}_{ee}^{-1}\mathbf{e}_t=\overleftarrow{\mathbf{p}}_t^T\Xi^T\Sigma_{ee}^{-1}\Xi\overleftarrow{\mathbf{p}}_t\sim\frac{(k+p)(n^2-1)}{n(n-(k+p))}F_\alpha(k+p,n-(k+p)) \tag{17}$$

where $\mathbf{S}_{ee}$ indicates a noise covariance matrix defined as $\mathbf{S}_{ee}=E(\mathbf{e}_t\mathbf{e}_t^T)$ and n denotes the number of training samples. The noise covariance matrix can be directly calculated with the covariance matrix of training samples and the noise extraction matrix.

$$\mathbf{S}_{ee}=E(\mathbf{e}_t\mathbf{e}_t^T)=\Xi E(\mathbf{p}_t^{t-l}\mathbf{p}_t^{t-l^T})\Xi^T \tag{18}$$

STATE SPACE MODEL UPDATING

1. Mean Vector, Covariance, and Correlation Matrix Updating

To develop a recursive approach of a state space model, the exponential weighted moving average [9,10] is adopted in this paper. The estimated mean vector at time point t is given as

$$\bar{\mathbf{z}}_t=(1-\alpha_t)\mathbf{z}_t+\alpha_t\bar{\mathbf{z}}_{t-1} \tag{19}$$

where $\bar{\mathbf{z}}_t$ and $\mathbf{z}_t$ denote the mean vector and a measured vector at time point t, respectively. α_t represents a forgetting factor at time point t. In general, process data are scaled by standard deviation. Thus, the standard deviations are updated as follows.

$$\mathbf{D}_t=(1-\beta_t)\cdot\mathrm{diag}(\tilde{\mathbf{z}}_t\tilde{\mathbf{z}}_t^T)+\beta_t\mathbf{D}_{t-1} \tag{20}$$

where $\mathbf{D}_t$ is a diagonal matrix whose elements are standard deviations a time point t, $\tilde{\mathbf{z}}_t=\mathbf{z}_t-\bar{\mathbf{z}}_t$, and β_t denotes a forgetting factor to update the standard deviations. With Eqs. (19) and (20), the covariance matrix at time t can be recursively calculated as

$$\Sigma_t=(1-\beta_t)\mathbf{D}_t^{-1/2}\tilde{\mathbf{z}}_t\tilde{\mathbf{z}}_t^T\mathbf{D}_t^{-1/2}+\beta_t\Sigma_{t-1} \tag{21}$$

where Σ_t is a correlation matrix at time point t.

2. Cholesky Factor Updating

CVA-based state space model is identified by using the past canonical variables as the approximated state. The state estimation matrix is obtained by the singular value decomposition (SVD) of the scaled Hankel matrix. The scaled Hankel matrix consists of the squared root of inverse matrices of the past and future covariance matrix.

Actually, it is not easy to recursively compute the squared root of covariance matrix at time t using the following relations:

$$\Sigma_t^{-0.5}=((1-\beta_t)\breve{\mathbf{z}}_t\breve{\mathbf{z}}_t^T+\beta_t\Sigma_{t-1})^{-0.5} \tag{22}$$

where $\breve{\mathbf{z}}_t=\mathbf{D}_t^{-1/2}\tilde{\mathbf{z}}_t$.

Since the Cholesky factor based scaled Hankel matrix can also be used to identify the state space model, the Cholesky factor updating [11] provides a clue of the scaled Hankel matrix updating. That is, Σ_t is factorized as $\mathbf{R}_t^T\mathbf{R}_t$ and Eq. (22) can be replaced as

$$\mathbf{R}_t^T\mathbf{R}_t=(1-\beta_t)\breve{\mathbf{z}}_t\breve{\mathbf{z}}_t^T+\beta_t\mathbf{R}_{t-1}^T\mathbf{R}_{t-1} \tag{23}$$

Eq. (23) can be differently expressed as follows.

$$\mathbf{R}_t^T\mathbf{R}_t=\beta_t^{0.5}\mathbf{R}_{t-1}^T(\mathbf{I}+\mathbf{a}_t\mathbf{a}_t^T)\mathbf{R}_{t-1}\beta_t^{0.5}=\beta_t^{0.5}\mathbf{R}_{t-1}^T\mathbf{K}_t^T\mathbf{K}_t\mathbf{R}_{t-1}\beta_t^{0.5} \tag{24}$$

where $\mathbf{a}_t=(1-\beta_t)^{0.5}\beta_t^{-0.5}\mathbf{R}_{t-1}^{-T}\breve{\mathbf{z}}_t=[\mathrm{a}_{t,1}\ \ \mathrm{a}_{t,2}\ \ \cdots\ \ \mathrm{a}_{t,h}]^T$. And $\mathbf{K}_t$ denotes the Cholesky factor of $\mathbf{I}+\mathbf{a}_t\mathbf{a}_t^T$. Thus, the Cholesky factor and the inverse Cholesky factor are recursively calculated as follows:

$$\mathbf{R}_t=\beta_t^{0.5}\mathbf{K}_t\mathbf{R}_{t-1} \tag{25}$$

$$\mathbf{R}_t^{-1}=\beta_t^{-0.5}\mathbf{R}_{t-1}^{-1}\mathbf{K}_t^{-1} \tag{26}$$

The computations of $\mathbf{K}_t$ and $\mathbf{K}_t^{-1}$ are given as

$$(\mathbf{K}_t)_{ij}=\begin{cases}(\mathrm{a}_{t,i}\mathrm{a}_{t,j})/(\delta_{t,j-1}\delta_{t,j}) & \mathrm{i<j}\\ \delta_{t,i}/\delta_{t,i-1} & \mathrm{i=j}\end{cases} \tag{27}$$

$$(\mathbf{K}_t^{-1})_{ij}=\begin{cases}-(\mathrm{a}_{t,i}\mathrm{a}_{t,j})/(\delta_{t,j-1}\delta_{t,j}) & \mathrm{i<j}\\ \delta_{t,i-1}/\delta_{t,i} & \mathrm{i=j}\end{cases} \tag{28}$$

where $\delta_{t,0}=1$, $\delta_{t,s}=\sqrt{1+\mathrm{a}_{t,1}^2+\cdots\mathrm{a}_{t,s}^2}$ for s=1, ⋯, h and $(\mathbf{K}_t)_{ij}$ and $(\mathbf{K}_t^{-1})_{ij}$ are the ij entries in $\mathbf{K}_t$ and $\mathbf{K}_t^{-1}$, respectively.

3. State Estimation and Noise Extraction Matrix Updating

The state estimation matrix at time t can be obtained from results of SVD of the scaled Hankel matrix based on the Cholesky factor at time t. Updating rules of Eqs. (21) and (26) provide the following recursive form of the scaled Hankel matrix:

$$\mathbf{R}_{ff,t}^{-T}\Sigma_{fp,t}\mathbf{R}_{pp,t}^{-1}$$
$$=\mathbf{K}_{ff,t}^{-T}\mathbf{R}_{ff,t-1}^{-T}((1-\beta_t)\beta_t^{-1}\mathbf{D}_{ff}^{-1/2}\tilde{\mathbf{f}}_t^{t+l+1}\tilde{\mathbf{p}}_t^{t-l^T}\mathbf{D}_{pp}^{-1/2}+\Sigma_{fp,t-1})\mathbf{R}_{pp,t-1}^{-1}\mathbf{K}_{pp,t}^{-1} \tag{29}$$

By applying SVD to Eq. (29), the state estimation matrix at time t, $\mathbf{J}_{k,t}$, can be calculated. After $\mathbf{J}_{k,t}$ is obtained, the sub-matrices at time point t, $\mathbf{A}_t$ and $\mathbf{C}_t$, can be expressed as follows:

$$\begin{bmatrix}\mathbf{A}_t\\ \mathbf{C}_t\end{bmatrix}=\mathrm{E}\begin{bmatrix}\mathbf{J}_{k,t}\mathrm{E}(\mathbf{p}_{t+1}\mathbf{p}_t^T)\mathbf{J}_{k,t}^T\\ \mathrm{E}(\mathbf{y}_t\mathbf{p}_t^T)\mathbf{J}_{k,t}^T\end{bmatrix}=\mathrm{E}\begin{bmatrix}\mathbf{J}_{k,t}\overleftarrow{\mathbf{R}}_{pp,t}^{t+1^T}\overleftarrow{\mathbf{R}}_{pp,t}^{t}\mathbf{J}_{k,t}^T\\ \overleftarrow{\mathbf{R}}_{pp,t}^{y(t)^T}\overleftarrow{\mathbf{R}}_{pp,t}^{t}\mathbf{J}_{k,t}^T\end{bmatrix}$$

$$\overleftarrow{\mathbf{R}}_{pp}=[\overleftarrow{\mathbf{R}}_{pp,t}^{y(t)}\ \ \overleftarrow{\mathbf{R}}_{pp,t}^{t}]=[\overleftarrow{\mathbf{R}}_{pp,t}^{t+1}\ \ \overleftarrow{\mathbf{R}}_{pp,t}^{y(t-1)}] \tag{30}$$

$$(\overleftarrow{\mathbf{R}}_{pp,t}^{t},\ \overleftarrow{\mathbf{R}}_{pp,t}^{t+1}\in\Re^{p\cdot(l+1)\times p\cdot l}\ \text{and}\ \overleftarrow{\mathbf{R}}_{pp,t}^{y(t)},\ \overleftarrow{\mathbf{R}}_{pp,t}^{y(t-l)}\in\Re^{p\cdot(l+1)\times p}.)$$

where $\overleftarrow{\mathbf{R}}_{pp}$ is the Cholesky factor of the covariance matrix of expanded past stacked vector at time t, $\overleftarrow{\Sigma}_{pp}=\mathrm{E}(\overleftarrow{\mathbf{p}}_t\overleftarrow{\mathbf{p}}_t^T)$. Current state sub-matrices, $\mathbf{A}_t$ and $\mathbf{C}_t$, can be recursively obtained by using the Cholesky factor updating of the covariance matrix of expanded past stacked vector.

4. Forgetting Factor Updating

For more effective state space updating, Choi et al. [10] proposed the forgetting factor updating method.

$$\alpha_t=\alpha_{max}-(\alpha_{max}-\alpha_{min})[1-\exp\{-\xi(\|\Delta\bar{\mathbf{z}}_{t-1}\|/\|\Delta\bar{\mathbf{z}}_{nor}\|)^n\}] \tag{31}$$

$$\beta_t=\beta_{max}-(\beta_{max}-\beta_{min})[1-\exp\{-\xi(\|\Delta\mathbf{M}_{t-1}\|/\|\Delta\mathbf{M}_{nor}\|)^n\}] \tag{32}$$

where α_{max}, β_{max}, α_{min}, and β_{min} are the maximum and minimum forgetting values, respectively, ξ and n are function parameters. $\|\Delta\bar{\mathbf{z}}\|$ is the Euclidean vector norm of the difference between two consecutive mean vectors, and $\|\Delta\mathbf{M}\|$ is the Euclidean matrix norm of the difference between two consecutive correlation matrices. Here, $\|\Delta\bar{\mathbf{z}}_{nor}\|$ and $\|\Delta\mathbf{M}_{nor}\|$ are the averaged $\|\Delta\bar{\mathbf{z}}\|$ and $\|\Delta\mathbf{M}\|$ obtained by using historical data. Six variables in Eqs. (31) and (32) are user-defined values.

ADAPTIVE MONITORING STATISTICS

CVA-based monitoring statistics are based on Hotelling's T^2 statistics. If it is assumed that these are normally distributed and temporally independent, Hotelling's T^2 can be replaced with the chi-squared distribution with equivalent confidence level [10,12]. Thus, adaptive monitoring statistics in the state and noise space can be determined as

$$\mathrm{T}_s^2=\mathbf{m}_t^T\mathbf{m}_t=\mathbf{p}_t^T\mathbf{J}_{k,t}^T\mathbf{J}_{k,t}\mathbf{p}_t\sim\chi_\alpha^2(\mathrm{k}_t) \tag{33}$$

$$\mathrm{T}_{wv}^2=\mathbf{e}_t^T\mathbf{S}_{ee,t}^{-1}\mathbf{e}_t=\overleftarrow{\mathbf{p}}_t^T\Xi_t^T\Sigma_{ee,t}^{-1}\Xi_t\overleftarrow{\mathbf{p}}_t\sim\chi_\alpha^2(\mathrm{k}_t+\mathrm{p}) \tag{34}$$

where k_t denotes state order at time t. Ξ_t indicates updated noise extraction matrix at time t defined as follows;

$$\Xi_t\equiv\begin{bmatrix}\dfrac{[\mathbf{J}_{k,t}\ \ \mathbf{0}]-[\mathbf{0}\ \ \mathbf{A}_t\mathbf{J}_{k,t}]}{[\mathbf{I}\ \ \mathbf{C}_t\mathbf{J}_{k,t}]}\end{bmatrix} \tag{35}$$

and $\Sigma_{ee,t}=\Xi_t\mathbf{R}_{pp,t}^T\mathbf{R}_{pp,t}\Xi_t^T$.

SIMULATION STUDIES

The proposed adaptive monitoring technique is applied to simulators of the CSTR with first order reaction [13] and the TEP [14].

1. CSTR Process

Two scenario-based process data sets are generated. Each data set consists of 1,000 training sample data and 1,000 test sample data, and test data are obtained from transient time-varying operating condition with the poisoning of the overall heat transfer coefficient. It is assumed that the poisoning of the overall heat transfer coefficient is normal state. Thus, process operating conditions of whole test data are normal. Scenarios are illustrated in Table 1.

Four time-lags and leads were selected for CVA modeling and the default values of Eq. (31) and (32) are $\alpha_{max}=\beta_{max}=0.99$, $\alpha_{min}=\beta_{min}=0.90$, $\xi=-\log(0.9)$, and n=1.3. The training samples are divided into two data sets with 500 samples. In first data set, the initial mean vec-

Table 1. Illustrations of Simulated CSTR scenarios

Scenario	Transition type	ts	Process condition
1	d(UA)/dt=−0.0005	0	normal
2	Tset: 368 °C→370 °C	200	normal
	d(UA)/dt=−0.0005	0	normal

UA: overall heat transfer coefficient, Tset: set point of temperature, ts: starting time of transient state

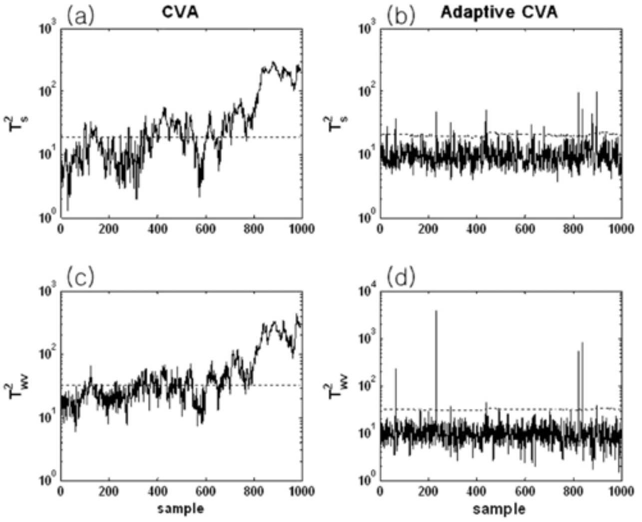

Fig. 1. Monitoring results for the CSTR scenario 1 (CVA vs adaptive CVA).

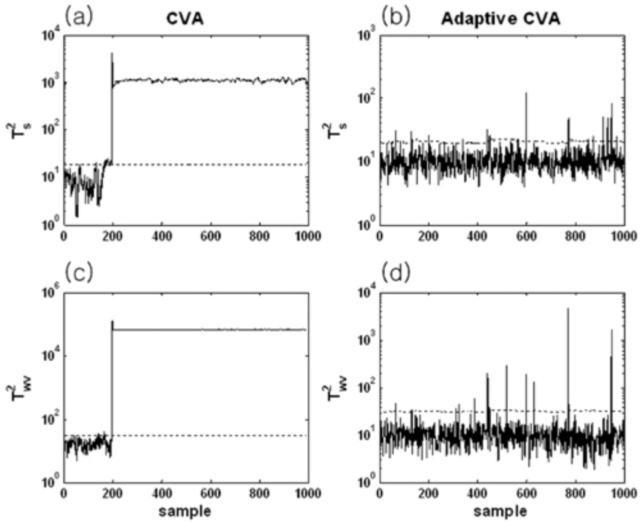

Fig. 3. Monitoring results for the CSTR scenario 2 (CVA vs adaptive CVA).

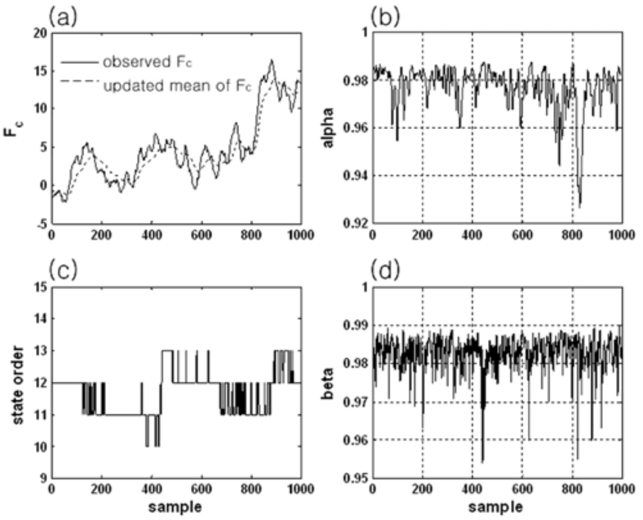

Fig. 2. Adaptive information of the CSTR scenario 1.

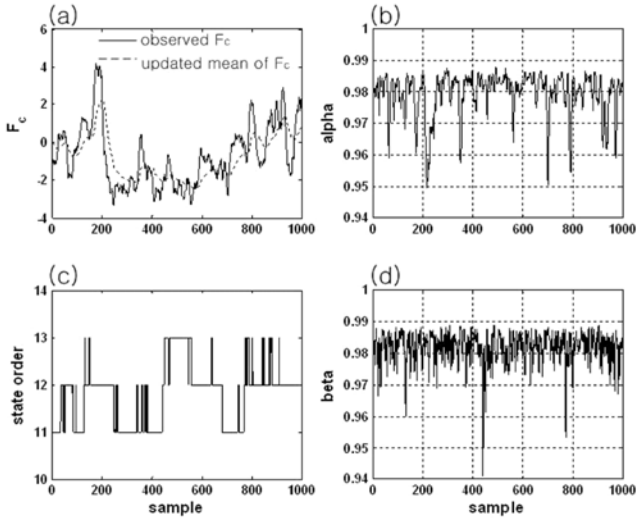

Fig. 4. Adaptive information of the CSTR scenario 2.

tor ($\bar{\mathbf{z}}_t=[\bar{\mathbf{f}}_t^T\bar{\mathbf{p}}_t^T]^T$) and the scaled Hankel matrix ($\mathbf{M}=\mathbf{R}_{ff,t}^{-T}\Sigma_{fp,t}\mathbf{R}_{pp,t}^{-1}$) are calculated. In the second set, the mean vector and the scaled Hankel matrix are updated with constant $\alpha=\beta=0.98$. And then, the averaged $\|\Delta\bar{\mathbf{z}}\|$ and $\|\Delta\mathbf{M}\|$ are calculated as $\|\Delta\bar{\mathbf{z}}_{nor}\|$ and $\|\Delta\mathbf{M}_{nor}\|$. And in the CVA model, the state order, k, was selected so that over 90% of the total variance of the scaled Hankel matrix was explained. Using the calculated initial and averaged values, the proposed adaptive method is applied to test data.

Fig. 1 shows the results of the static and the adaptive CVA based monitoring for the CSTR scenario 1. Figs. 1(a) and 1(c) indicate the static CVA-based monitoring charts in the state space and noise space, respectively. Figs. 1(b) and 1(d) are the adaptive CVA-based monitoring charts in the state and noise space, respectively. The proposed approach effectively updates the model for fault detection under time-varying operating condition.

The updating information of the CSTR scenario 1 in Table 1 is shown in Fig. 2. The solid line in Fig. 2(a) indicates the observed value of coolant flowrate. The dotted line in Fig. 2(a) shows the estimated mean of that using the forgetting factor updating (see Fig. 2(b)) of the mean vector for changes of operating condition. Fig. 2(c) indicates the change of state order which explains over 90% of total variance of scaled Hankel matrix. Also, the forgetting factor of the Hankel matrix is appropriately updated against the change of operating state (see Fig. 2(d)).

The implementation results of the CSTR scenario 2 are shown in Figs. 3 and 4. Test data are generated under time-varying and transient operating conditions, which are the poisoning of the overall heat transfer coefficient and temperature set point change. Fig. 3 shows the results of static and adaptive CVA-based monitoring, and the adaptive information for scenario 2 are represented in Fig. 4. Such as scenario 1, the proposed approach shows good adaptation performance for time-varying with transient operating condition.

2. Tennessee Eastman Process (TEP)

The data set of various faults and transient states for TEP can be downloaded from the website, http://brahms.scs.ucui.edu/. Two set point changed data sets among them were selected, which were regarded as normal operating condition. The information of the selected transient states is shown in Table 2. Also, the default values of Eqs. (31) and (32) were $\alpha_{max}=\beta_{max}=0.99$, $\alpha_{min}=\beta_{min}=0.90$, $\xi=-\log(0.9)$, and n=1.3 such as the CSTR application. To build the CVA model, two time-lags and leads were selected. The 980 training samples were divided into two data sets with 460 samples. $\|\Delta\bar{\mathbf{z}}_{nor}\|$ and $\|\Delta\mathbf{M}_{nor}\|$ and were obtained from the second training data set and the number of state order, k which captures over 95% of total variance of the Hankel matrix at time t.

Table 2. Illustrations of Simulated TEP scenarios

Scenario	Transition type	ts	Process condition
1	SPCh of A/C feed ratio [IDV(1)]	150	normal
2	SPCh of T_{CW} [IDV(5)]	150	normal

SPCh: set point change, T_{CW}: condenser cooling water inlet temperature, ts: starting time of transient state

Figs. 5(a) and (c) show the monitoring results from the conventional CVA for the TEP scenario 1. The TEP scenario 1 is illustrated in Table 2 and is regarded as a normal operating condition. Though the process was normally operated, the static approach did not provide correct information of operating conditions. However, the results of the adaptive CVA in Figs. 5(b) and (d) show that the proposed approach can appropriately update the CVA model under the transient state. Fig. 6 illustrates the updating information about mean, updating parameters, α and β, and selected state order at every updating time. The monitoring results for the other scenario in Table 2 are shown in Fig. 7. Such as the TEP scenario 2, the adaptive CVA provides the appropriate updating model for fault detection. Related

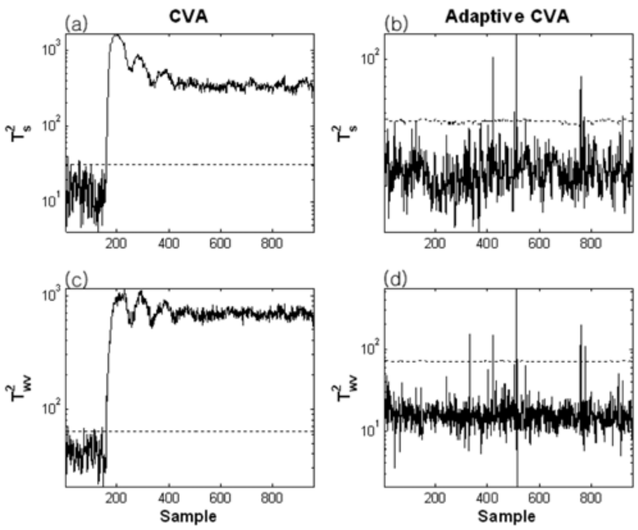

Fig. 5. Adaptive information of the TEP scenario 1 (CVA vs adaptive CVA).

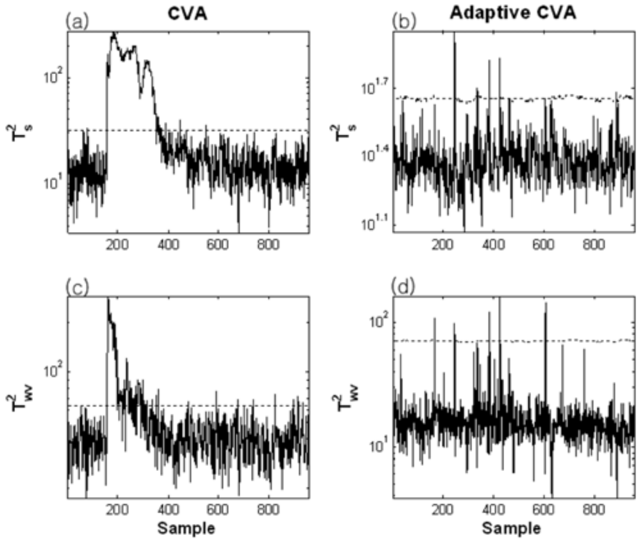

Fig. 7. The monitoring results for the TEP scenario 2 (CVA vs adaptive CVA).

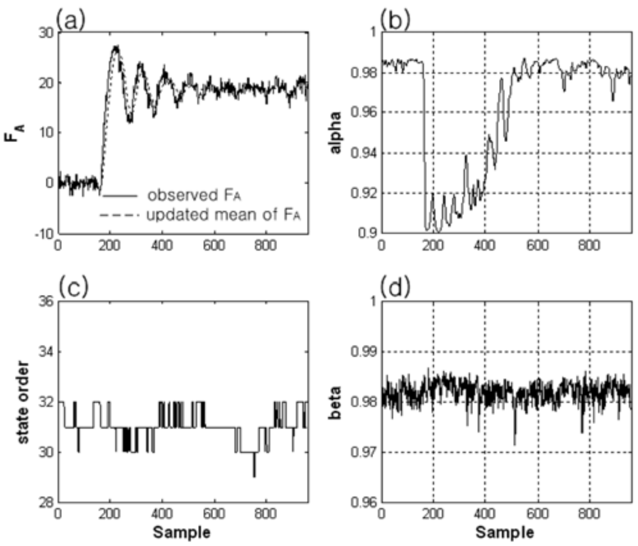

Fig. 6. Adaptive information of the TEP scenario 1.

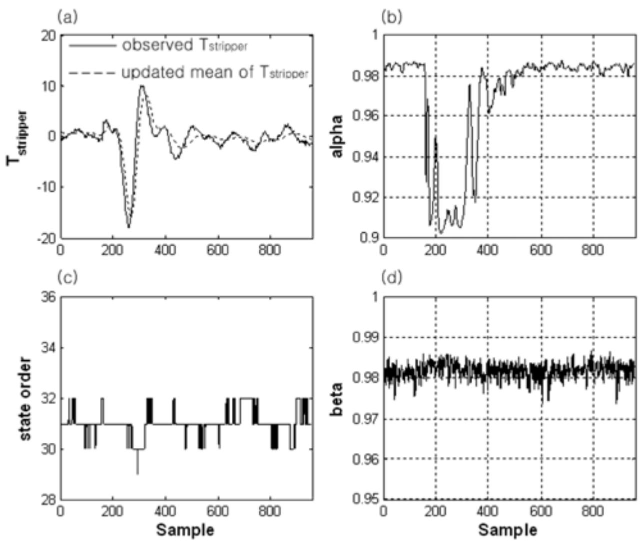

Fig. 8. Adaptive information of the TEP scenario 2.

updating information is depicted in Fig. 8.

CONCLUSIONS

This paper focuses on the state space model-based adaptive monitoring technique using recursive CVA. To formulate the recursive scheme of state space model, CVA-based state space modeling method and the recursive Cholesky factor updating are employed. The adaptation of the proposed method is verified through application to the CSTR and the TEP simulator under time-varying and transient process operating condition. Actually, it is not efficient to gather normal operating data and build a model for every operating state. Though the proposed approach cannot distinguish transient or time-varying process conditions from process abnormalities, the proposed algorithm provides a suitable state space model for fault detection without gathering normal operating data after operating condition change. As a future work, a reasonable criterion will have to be developed that not only our proposed but the other adaptive approaches are applied to transient and time-varying processes.

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