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An Experimental and Theoretical Study on the Initial Period of Cake Filtration

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Abstract−Experimental measurements were made of the average specific cake resistance during the initial period of cake filtration, and the theoretical calculations about the period were also performed. The "filtration-permeation method" in the filtration cell of small area was used to measure the flow rate during the initial period of filtration, which is essentially characterized by the large flux due to fast flow rate and the rapid change of flow rate within a relatively short time interval. The measured average specific cake resistances of thin cakes which represent the cakes of initial period had very large values compared to the overall average specific cake resistance. This experimental result was contrary to the conventional theory about the initial period. Applying the "unified theory on solid-liquid separation" to the initial period, the average specific cake resistances at the initial period can have the large values--more than two times greater than that of the overall value.

Key words: Cake Filtration, Initial Period of Cake Filtration, Filtration-Permeation

INTRODUCTION

The theory of cake filtration has developed significantly during the past 50 years. We now have knowledge about the distribution of pressure and porosity in a filter cake, about filtration with very compressible cake, about dense skin and also about filtration with sedimentation. But due to experimental and theoretical difficulties, the knowledge about the phenomena during the initial period of filtration developed slowly. As experimental results during the initial period were not available, the influence of the initial period on the entire filtration was very difficult to determine.

The so-called parabolic equation, which was proposed by Sperry [1917] and modified by Carman [1938] and used in plant operations, is good for most cake filtration operations. The average specific cake resistance is usually determined with this equation by measuring filtrate volume with respect to filtration time. The measurement of the exact filtrate volume at the initial period is very difficult. As a result, experimental study of the initial period of filtration was not possible before the present study.

Carman [1938] assumed that the resistance of filter medium increases at the initial period by the clogging of filter medium during the formation of filter cake, and thereafter the increment of filter medium resistance ceases after the formation of filter cake. Tiller [1960] calculated the initial average specific cake resistance by a theoretical analysis for the phenomena during initial period. The calculated value of the average specific cake resistance at the initial period was smaller than that at the latter period. Tiller [1990] analyzed an accurate experimental filtration result using two to eight points of data from the beginning, and he concluded that the initial average specific cake resistance was small but the resistance of filter medium at the initial period was large.

In this study, an experimental method, "filtration-permeation," developed by the author [1990] was applied. By this method the average specific cake resistance can be measured not only with the filtration data but also with the permeation data. The average specific cake resistance for very thin cakes was measured by the "filtration-permeation" method. The average specific cake resistances at the initial period were calculated with the "unified theory on solid-liquid separation" which was also developed by the author [Yim, 1996; Yim et al., 1997].

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THEORY

1. Analysis of Filtration Procedure by the Conventional Notion

The equation representing the cake filtration originated from Darcy's equation [1856], and now the following form of equation is used.

$$\frac{dV}{dt}=\frac{\Delta P}{\mu(\alpha_{av}W+R_m)} \tag{1}$$

At constant pressure filtration, it is normally considered that the pressure drop across the cake and filter medium ΔP and the viscosity of liquid μ are constant. The separation of solid particles in suspension is performed at the cake surface, so the resistance of a filter medium R_m remains constant during cake filtration if there is no migration of small particles in the cake. This subject will be discussed later.

The value of average specific cake resistance α_{av} remains constant during the entire filtration procedure by conventional cake filtration theory. Thus, the one variable for cake filtration of constant pressure is the mass of solid in filter cake per unit filtration area W in Eq. (1). At the moment of starting filtration, there is no cake on a filter medium. The mass of cake increases as filtration proceeds, and this decreases the rate of filtrate dV/dt by Eq. (1). Eq. (1) does not give any clue for analyzing the initial period of filtration. With this equation, there is no difference between the cake of the initial period and that of the latter period.

2. Theoretical Prediction of Average Specific Cake Resist-

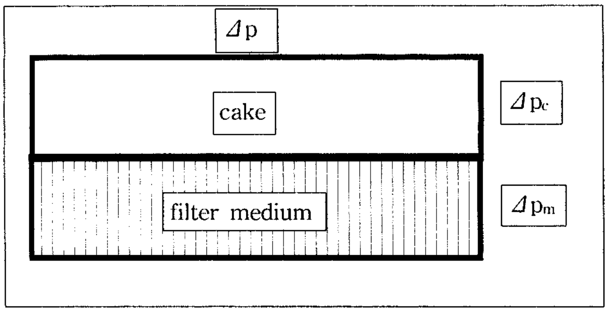

Fig. 1. The pressure drops in cake filtration.

ance during Initial Period by Tiller

Tiller noted that the ratio of α_{av} W and R_m in Eq. (1) is very different between the initial and latter period of filtration. At the moment of starting filtration, all of the pressure drop for filtration ΔP acts through the filter medium only as there is no filter cake. After that moment, a very thin cake as shown in Fig. 1 is formed and the pressure drop for filtration ΔP is divided into the pressure drop across the filter medium and across the filter cake. Finally, the value of α_{av} W becomes much greater than that of R_m, and almost of all pressure drop ΔP acts through the cake. During all of the filtration procedure, overall pressure drop ΔP is the sum of the pressure drop across the cake ΔP_c and across the filter medium ΔP_m as shown in Eq. (2).

$$\Delta p_c + \Delta p_m = \Delta p \tag{2}$$

The summation of the above is as follows. The overall pressure drop ΔP is constant during all of the filtration procedure for constant pressure filtration. At the start of filtration, ΔP_c is zero and ΔP_m is ΔP. And ΔP_c increases nearly to ΔP, and ΔP_m decreases nearly to zero within a relatively short period.

Ruth [1946] invented an apparatus called a compression-permeability cell (CPC) with which theoretical study for the internal cake structure became possible. The CPC gives the relation between the specific cake resistance and solid compressive pressure as below.

$$\alpha = a p_s^n \tag{3}$$

The values of 'a' and 'n', which are determined by the CPC data, represent the characteristics of a cake. We call the value of 'n' the compressibility of a cake. The P_s in the equation, which is the solid compressive pressure, is the pressure applied on the solid particles of cake.

The α in the equation is not the average specific cake resistance α_{av} measured by experimental cake filtration, but the specific resistance of a cake which has uniform porosity. Uniform porosity cannot be obtained by conventional filtration except for an incompressible cake, which is very rare for plant operation.

The relation between α_{av} and α is conventionally expressed by Eq. (4) which has been used since the early 1950's to calculate the average specific cake resistance with CPC data.

$$\alpha_{av} = \frac{\Delta p_c}{\int_0^{\Delta p_c} \frac{dp_s}{\alpha}} \tag{4}$$

Eq. (5) is obtained by substituting α of Eq. (3) into Eq. (4).

$$\alpha_{av} = \frac{\Delta p_c}{\int_0^{\Delta p_c} \frac{dp_s}{a p_s^n}} = a(1-n)\Delta p_c^n \tag{5}$$

The above equations, which are the basic concepts for cake filtration, have been established since 1960 when Tiller analysed the phenomena about the initial period of filtration.

The analysis of Tiller [1960, 1990] about the initial period was based on Eq. (5). As mentioned above, the ΔP_c at the beginning of filtration is zero, and it increases to ΔP according to the thickness of cake. Thus, the value of ΔP_c at the beginning is null.

The value of α_{av} calculated by Eq. (5) with small ΔP_c is smaller than that with large ΔP_c. Based on this calculated result, Tiller concluded that the average specific cake resistance α_{av} at the initial period of filtration is smaller than that at the latter period.

Tiller [1990] analyzed accurate experimental filtration results by Hosseini [1977] to prove his calculated result. On the contrary, the experimental results show that the combined resistance of the cake and medium at the initial period is larger than the calculated values. He concluded that the medium resistance increases at the initial period to a great extent, so the average specific cake resistance could be small at the period. But the experimental result of Hosseini [1977], which had only two initial data points, shows merely slight increase of the combined resistance of the cake and filter medium at the initial period. The two initial data points were in a region that is very difficult to measure.

3. Theoretical Prediction of Average Specific Cake Resistance during Initial Period by the Unified Theory on Solid-Liquid Separation

The author [1996, 1997] presented the "unified theory on solid-liquid separation" which can describe the process of cake filtration, cake expression, hindered sedimentation, and centrifugal filtration. The unified theory is based on Darcy's equation and also the idea that the solid compressive pressure on a cake surface is not null but has different values according to different solid-liquid separation operations.

At the initial period of filtration, the rate of flow is sufficiently fast for exerting large drag force on the particles situated at the cake surface. The boundary conditions of a cake from zero to ΔP_c in Eq. (4) are not valid in this case. The particles of the cake surface receive a certain value of solid compressive pressure $p_{s,u}$ instead of zero.

Then Eq. (4) becomes:

$$\alpha_{av} = \frac{\Delta p_c}{\int_{p_{s,u}}^{\Delta p_c} \frac{dp_s}{\alpha}} = \frac{\Delta p_c}{\int_{p_{s,u}}^{\Delta p_c} \frac{dp_s}{a p_s^n}} = \frac{a(1-n)\Delta p_c}{\Delta p_c^{1-n} - p_{s,u}^{1-n}} \tag{6}$$

We use the same notion that the ΔP_c is small at the initial period of filtration. But the notion of the solid compressive pressure of the filter cake surface $p_{s,u}$ is suggested. By this new idea, the cake at the initial period may have smaller porosity than at the latter period, because of the compression of the cake due to the large drag force from the cake surface by rapid flow rate. A small diminution of cake porosity gives a great enlargement of average specific cake resistance. This phenomenon is included in Eq. (6). In conclusion, the value of average specific resistance α_{av} could be increased even though the ΔP_c is small at the initial period of filtration.

4. Method of Calculation

As Tiller [Tiller et al., 1960; Tiller, 1990] did not clearly describe his calculating method of the average specific cake resistance at the initial period, the method presented below is based on his concept.

At the moment of starting filtration there is no cake at all, so the pressure for filtration is applied only on the filter medium. The velocity of liquid which passes through filter medium is maximum at the moment, because the value of cake mass W in Eq. (1) is zero. The velocity of filtrate, dV/dt can be calculated with the medium resistance R_m. After this moment, cake is formed corresponding to the volume of filtrate that passed through the filter medium. A very little amount of cake is formed after a very short period of time.

The resistance of a cake at the initial period could be smaller than that of the filter medium. And the pressure drop across the filter cake is smaller than that across the filter medium. The average specific cake resistance at the moment can be calculated by Eq. (5) with an arbitrary small pressure Δp_c. To determine the cake mass W, Eqs. (7) and (8) are adopted by the conception of Eq. (1).

$$\frac{dV}{dt}=q=\frac{\Delta p_c}{\mu\alpha_{av}W} \tag{7}$$

$$\frac{dV}{dt}=q=\frac{\Delta p-\Delta p_c}{\mu R_m} \tag{8}$$

This and following concepts are developed in this study. The flow rate dV/dt in the above equations means the flow rate of the filtrate passed after the filter medium, i.e., the velocity at the empty tower V_o.

The same value of dV/dt must be obtained by Eq. (1) which comprises the cake and filter medium, and by Eq. (7) for only the pressure drop across the cake, and by Eq. (8) for the filter medium. Thus the mass of cake per unit filter area, W, can be expressed as below by Eqs. (7) and (8).

$$W=\frac{\Delta p_c R_m}{(\Delta p-\Delta p_c)\alpha_{av}} \tag{9}$$

The value of W is calculated with the average specific cake resistance. This means that the average specific cake resistance at any cake mass per unit filter area can be obtained by varying the value of W. The above procedure is performed as the calculation with the unified theory. The flow rate of filtrate dV/dt at the start of filtration is calculated by Eq. (1) assuming no cake. With this flow rate, the drag force acting on the first solid layer is determined.

We assumed that the particles are spheres, which could be justified if the value of the $N_{Re,p}$ is smaller than 50 [McCabe et al., 1995]. The average particle diameter 10.6 μm determined by Malvern mastersizer was used. With these values, the value of $N_{Re,p}$ can be calculated. The $N_{Re,p}$ has a maximum value at the very first moment, and thereafter it becomes smaller by the decreasing flow rate according to increasing cake mass. Eq. (10) can be adopted to know the drag force.

$$F_D=C_D A_p \rho\frac{u^2}{2}=\frac{24}{N_{Re,p}}A_p\rho\frac{u^2}{2} \tag{10}$$

The drag force pushes the solid layers situated below. The solid compressive pressure (P_s) of the first solid layer is determined with the drag force and the porosity of first solid layer. It is the $p_{s,u}$ in Eq. (6), and with it we calculate the average specific cake resistance. The $N_{Re,p}$ calculated without cake is 0.5, i.e. less than 1, so Eq. (10) can be applied to all of the filtration procedure.

5. Definition of the Initial Period of Filtration

In the conventional concept of filtration, the average specific cake resistance is constant during the entire filtration procedure; thus, there is no difference between the initial period and thereafter. By Tiller's analysis, the average specific cake resistance at the initial period is somewhat smaller than that of the latter period. But with the theoretical result of the 'unified theory for solid-liquid separation' proposed by the author, the average specific cake resistance at the initial period could have larger values than the latter period. The initial period in this paper means the period during which the average specific cake resistance is different from the latter period. According to an example presented by Tiller et al. [1960], the initial period was within two minutes after the start of filtration operation.

EXPERIMENTAL

1. Difficulties in the Measurement of Experimental Data during the Initial Period of Filtration

Initially, there is no cake upon the filter medium, and in the succeeding period there is only a small amount. The overall resistance at this period is small, so the rate of flow is very fast compared with the latter period. It is impossible to measure the exact flow rate at any instant in this period, because the fast flow through the filter medium leaves the particles upon filter medium as a cake, and the formed cake reduces the flow rate rapidly. The variation of flow rate is too fast to be measured.

This phenomenon has an influence on the measurement in real filtration experiments as follows. As the fast flow rate creates a surge to the surface of the liquid in the receiver of filtrate, it is difficult to measure the filtrate volume precisely. The initial period is within one or two minutes, so it is very difficult to measure the variation of the rate of flow accurately. There were many trials to measure the variation of the rate of flow using a combination of the digital chemical balance and computer, but the exact measurement was always hindered by the rolling of water upon a balance.

The difficulties in measuring the flow rate during initial period of filtration can be summarized as the large amount of filtrate due to fast rate of flow caused by thin cake, the rapid change of the rate of flow due to the fast formation of cake, and finally the short duration of the initial period.

2. Experimental Measurement during Initial Period with "Filtration-Permeation" Method

The author proposed an experimental technique termed "filtration-permeation" to study cake filtration. With this technique, we could determine the mass fraction of cake for a floc filtration [Yim, 1990a], the compressibility of a cake with only one filtration-permeation experiment by stepwise pressure variation during the permeation period [Yim, 1990a], the average specific cake resistance of materials very hard to filter [Yim, 1990b], and the phenomena about the filtration with sedimentation [Yim, 1999].

The procedure of "filtration-permeation" is that conventional filtration is performed during the frist half of the procedure followed by the permeation period during which particle-eliminated water is permeating through the preformed filter cake. Average specific cake resistance during filtration period $\alpha_{av,f}$ is calculated using filtration data and Ruth's equation as conventional filtration.

During the permeation period, Eq. (1) is applied directly to the permeation data. The value of W, cake mass per unit filter area, is constant in the permeation period as all particles in suspension have already been changed into a cake. The pressure drop ΔP in constant pressure filtration, viscosity μ, and the resistance of filter medium R_m are also constant during the permeation period; thus the flow rate dV/dt must not change. With the flow rate dV/dt, average specific cake resistance during permeation $\alpha_{av,p}$ can be calculated with the equation. The $\alpha_{av,f}$ by filtration and $\alpha_{av,p}$ by permeation must coincide if there were no other additional phenomena such as sedimentation during filtration or fine particle migration through cake and/or filter media [Yim, 1999]. When sedimentation occurs at the filtration period, the two values obtained are different. In that case, the real value of solid mass fraction of suspension S which enters into the cake is changed by the sedimentation, and this value is very difficult to measure [Yim, 1999]. It is reasonable that $\alpha_{av,p}$ during permeation represent exact average specific cake resistance because there is no sedimentation during permeation.

In an ordinary Büchner funnel test, the filtration proceeds until all of the particles in suspension change into cake. The data for the initial period is very hard to obtain with this procedure. If we reduce the amount of the suspension to have a very thin cake which represents the initial period of filtration, the filtration is terminated before we have measured the first experimental datum with an ordinary Büchner funnel.

In this study, a thin cake was formed with small amount of suspension and at the same time particle-eliminated water was added into the cell. The particle-eliminated water passed through the preformed thin cake. The average specific cake resistance $\alpha_{av,p}$ can be measured with the permeating velocity dV/dt by the method mentioned already. By selecting the amount of suspension, a very thin cake with certain value of W can be formed and the average specific cake resistance can be measured during the permeation period. This experiment could be done for various values of cake mass per unit filter area W. This means that the average specific cake resistance at an instance during the initial period cannot be measured in conventional filtration because of the rapid change of the rate of flow, but it could be measured by "filtration-permeation" at any cake thickness.

To measure the average specific cake resistance during filtration is not possible also, because the filtration period is too short to measure the flow rate. But the permeation period can be prolonged as long as we want to measure the flow rate by adding the particle-eliminated water.

With a conventional Büchner funnel the mass flux at the initial period is too large to be measured, because the mass flux of permeate is very fast due to the small resistance of thin cake. The problem is settled by adopting a filtration cell of small area, i.e., filtration-permeation was performed in a cell of 1.5 cm diameter instead of the conventional 4 cm diameter cell.

According to Eq. (1), the flow rate dV/dt without cake is 5 cm/s through a filter medium which has the R_m of 10^9 m^{-1} at the pressure of 0.5 atm, and for the permeation with particle-eliminated water of 22 °C. The mass flux with conventional 4 cm filter cell which has the filter area of 12.6 cm^2 is 63 cm^3/s. In this case normal filtrate receiver of 400 cm^3 is filled up within 7 seconds during which exact measurement is not possible. The mass flux with 1.5 cm cell which has the filter area of 1.54 cm^2 is 7.7 cm^3/s, and the time to be filled up is more than 52 seconds. The measurements can be performed accurately to some degree at this maximum flow rate. Even if the thinnest cake is formed upon the filter medium the flow rate becomes slow, so the measurement can be made more accurately.

With "filtration-permeation" in a cell with small filter area, the three difficult experimental problems for the initial period of filtration can be settled.

4. Experimental Apparatus and Methods

A conventional Büchner funnel cell has an enlarged section to have large volume of suspension with the same height. Considerable amount of particles sediment upon the enlarged place. This portion of particles does not contribute to filtration. In this study, a cylindrical cell with constant diameter was used to prevent the above effect.

An aspirator for depressurizing the system always produces fluctuation of pressure which changes the structure of filter cake. A 40 liter air tank was installed between aspirator and filtration apparatus. Before the start of filtration the air tank was depressurized to a certain pressure with aspirator; the valve between tank and filtration unit was shut off during this operation. Then the valve between aspirator and tank was shut off, and the aspirator was stopped. Suspension was poured into the filtration cell, and filtration operation begun by opening the valve between the filter unit and the air tank. All of the filtration operation was performed by the pressure of tank.

The maximum volume of filtrate was 400 mL and the volume of air tank was 40 L; thus, the pressure variation between the start and the end of filtration was 1%. The error arising from the 1% pressure variation was included in the experimental result. As all of the filtration procedure was performed by the pressure in the air tank, a leak in the connections had to be absolutely prevented. Especially the connecting part of the support for filter paper and filtration cell was sealed with rubber band to prevent the inflow of air.

Calcium carbonate fabricated by Yakuri Pure Co. was used as filtering material. The volumetric average diameter of the particle was 10.6 μm measured by Malvern mastersizer, and 90% of particle weight was larger than 1.0 μm. The calcium carbonate was kept at 105 °C. At every test, a part of the material was taken out and cooled in a desiccator during 30 minutes, and then the weight was measured. The aim of the above procedure was to prevent the error due to humidity.

Toyo Adventec 5C composed of paper fiber and cellulose filter medium for micro-filtration [MF membrane] fabricated by Micro-filtration System Co. with pore sizes of 0.8 μm, 0.45 μm, and 0.1 μm were used. Conventionally, the characteristics of filtration could be changed with filter media [Yim, 1998], but there was not noticeable difference in experimental results between above filter media.

Water passed through reverse osmotic membrane was used as particle-eliminated water. The flow rates of particle-eliminated water through the filter medium were measured, and the rates of flow did not change for large amounts of particle-eliminated water passed through.

At experimentation, particle-eliminated water was added very carefully to the filter cell when about 80% of initial suspension had been filtered. Other experimental operations were performed as conventional filtration methods.

RESULTS AND DISCUSSION

1. The Results of Conventional "Filtration-Permeation"

A typical experimental result of "filtration-permeation" is presented in Fig. 2. The 200 mL calcium carbonate suspension of 1 wt% was filtered and permeated at 0.5 atm with the filter medium of MF membrane of 0.45 μm. The cake mass per unit filter area W was 1.44 kg/m^2 when all of the particles in suspension had been changed into cake.

Filtration proceeds in the first half of "filtration-permeation". In typical filtration the relation between Δt/ΔV and V is represented by a straight line with a certain slope. With the slope the average specific cake resistance can be calculated, and in this case the value is 5.62×10^{10} m/kg.

When all of the particles in suspension have been filtered, the particle-eliminated water passes through the preformed filter cake, and the black points almost parallel to the x-axis in the right side of Fig. 2 show this permeation period. The values of Δt/ΔV are nearly constant during this period, and this means that the permeation velocity (ΔV/Δt) is constant throughout the permeation. The average specific cake resistance calculated with the value of Δt/ΔV and Eq. (1) is 6.89×10^{10} m/kg. The difference of the values between filtration and permeation is mainly due to the sedimentation during filtration operation [Yim, 1999]. The value measured during filtration includes the effect of sedimentation, but this effect is excluded during permeation [Yim, 1990a]. It should be mentioned again that the difference in average specific cake resistance of such extent does not influence the analyses of the phenomena of the initial period of filtration.

The variation of experimental data at the permeation period is due to the measurement of time, i.e., the difference of 1 second gives a variation of points up and down as shown in the figure.

The constant flow rate during permeation means that the cake property is not changed by the operation of permeation. When the small particles in cake are washed by particle-eliminated water

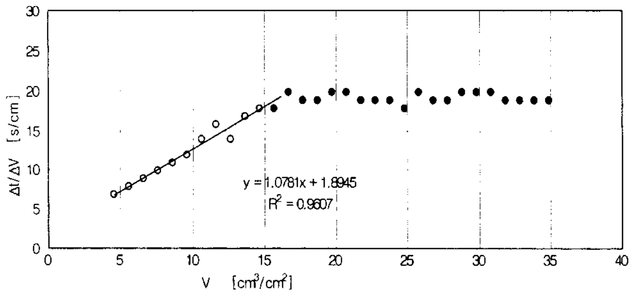

Fig. 2. Filtration-Permeation result of 1% $CaCO_3$ suspension (W=1.44 kg/m^2) with 0.45 μm MF membrane.

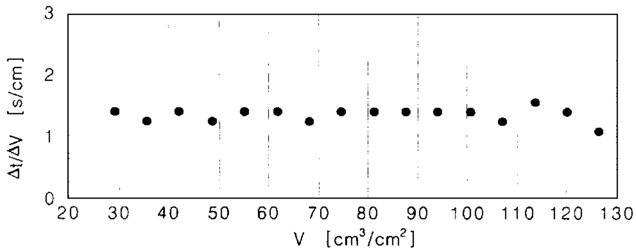

Fig. 3. Filtration-Permeation result of $CaCO_3$ suspension (W=0.0136 kg/m^2) at 0.5 atm with 0.8 μm MF filter of 1.54 cm^2.

through the cake and filter medium, the overall resistance and Δt/ΔV must decrease. If small particles in the cake migrate and stop at the narrows, the resistance and Δt/ΔV increase. We can conclude that the above two phenomena did not happen because the Δt/ΔV shows constant values during permeation.

2. Average Specific Cake Resistance by the "Filtration-Permeation" of Thin Cake

Fig. 3 shows the "filtration-permeation" results with a small quantity of suspension to make thin cake in a filter cell of small diameter.

The cake mass per unit filter area W is 0.0136 kg/m^2 which is 0.94% of the previous test. The filtration period is not shown in Fig. 3 because filtration had been finished before we measured the first experimental data. The average specific cake resistance can be calculated with the value of Δt/ΔV during permeation and Eq. (1). In conventional filtration the resistance of the filter medium is frequently ignored as cake resistance is much larger than the resistance of filter medium for almost all of the filtration procedure [Grace, 1953]. In our case the cake resistance with small mass is not large enough to ignore the filter medium resistance. The resistance of the filter medium was measured with particle-eliminated water. The value of R_m for MF membrane of 0.8 μm was 3.20×10^9 m^{-1}. The average specific cake resistance with the R_m and the value of Δt/ΔV is 1.91×10^{11} m/kg; this value is 2.8 times greater than the previous result measured for the conventional filtration.

3. Average Specific Cake Resistance at Initial Period Measured by "Filtration-Permeation"

Fig. 4 represents the average specific cake resistances obtained by the method already mentioned with varying the cake mass per unit filter area W. When W is greater than 0.1 kg/m^2 the average specific cake resistances are almost identical. The assumption that average specific cake resistance does not change during filtration [Carman, 1938] is valid for this range of W. It is also an experimental proof that Ruth's equation can be applied to almost all of

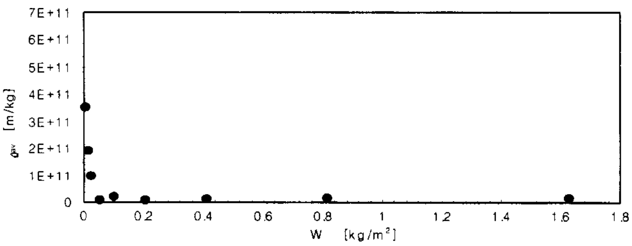

Fig. 4. Measured average specific cake resistances for the various W of $CaCO_3$ suspensions at 0.5 bar with 0.8 μm MF filter.

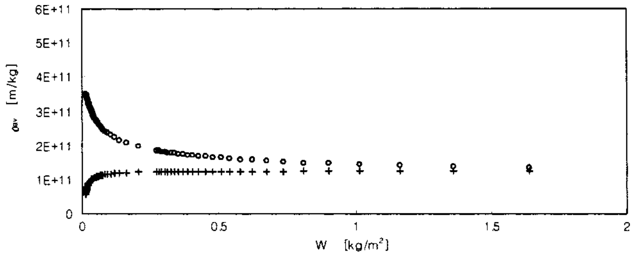

Fig. 5. Calculated average specific cake resistances for various W by the notion of Tiller (+) and Yim (o).

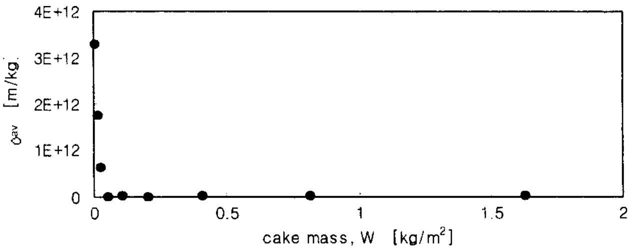

Fig. 6. Measured average specific cake resistances for the various W of $CaCO_3$ suspensions at 0.5 bar with 0.1 μm MF filter.

the filtration period.

When W is less than 0.1 kg/m^2 the average specific cake resistances have larger values. The minimum value of W in our experiment was 0.00635 kg/m^2 and the average specific cake resistance in this case was 3.56×10^{11} m/kg. This experimental result is opposite to the theory of Tiller [Tiller et al., 1960; Tiller, 1990], but coincides with the calculated result of the "unified theory on solid-liquid separation" [Yim et al., 1997].

4. Average Specific Cake Resistance in the Initial Period by Calculation

In Fig. 5 the calculated values of average specific cake resistances by the procedure already mentioned are plotted against the W.

The calculated results by the conception of Tiller shows that very thin cakes, i.e., cakes which have small values of W, have smaller values of average specific cake resistances. These calculated results express Tiller's concept exactly. He insists that the pressure drop of cake Δp_c is small at the initial period as the resistance of cake is small due to the small W, and the rest of the overall pressure drop acts across the filter medium. The small Δp_c yields the small average specific cake resistance by Eq. (5). But these calculated results are opposite to our experimental results.

The calculated results by the method proposed in this study do not coincide exactly with the experimental results, but there obviously exists a similarity between the two results. The maximum calculated average specific cake resistance is two times greater than that of the latter period.

The concept of our theory is as follows. At initial filtration period the pressure drop across a cake is small due to the pressure distribution with filter media, and this phenomenon can cause a reduction of the average specific cake resistance as Tiller indicates. But the fast rate of flow which is inevitable at the initial period gives more drag force to the particles composing the cake. The particles receive a not omittable drag force from the first solid layer, i.e. the first layer of cake next to suspension. By this drag force, the cake is compressed; compressed cake has smaller porosity, and smaller porosity enlarges the average specific cake resistance to a great extent. The above concepts are included in the calculations, and same concepts are used for the calculation of expression procedure and hindered sedimentation procedure in the "unified theory on solid-liquid separation".

The calculated result by Tiller's conception of the initial period could not yield larger values of average specific cake resistance than the latter period for other types of cakes also. But the calculated average specific cake resistance by our "unified theory on solid-liquid separation" can yield larger or smaller values according to the types of cakes and filtration conditions.

5. Average Specific Cake Resistances According to Filter Medium

It was assumed that filter medium did not have an influence on usual cake filtration. For properly selected filter media, practically the same values of average specific cake resistance were measured for various filter media [Yim et al., 1998]. But this is not justified for the initial period of filtration. We performed the same experiments with an MF filter of 0.1 μm and with the Toyo 5C and 5A filter papers.

The average specific cake resistances measured for various W with 0.1 μm MF filter are presented in Fig. 6. The tendency is not changed, but the average specific cake resistance for the smallest W is 3.30×10^{12} m/kg which is much greater than that with 0.8 μm MF filter. The average specific cake resistances for the large W with two different pore size filter media are practically the same. The resistance of the filter medium of the 0.1 μm MF filter was 9.10×10^{10} m^{-1} and that of 0.8 μm MF filter was 3.20×10^{9} m^{-1}. This difference is not a small one, but is natural if we take into account the difference of pore size.

The large difference in average specific cake resistance between the two filter media could not be explained with the previous conception. The diameter of calcium carbonate particles was from 0.3 μm to 50 μm. So there exists only a very small number of particles under 0.1 μm. The number of fine particles does not greatly affect the mass fraction; thus, it is possible that there exists enough fine particles for clogging inside the pores of the 0.1 μm MF filter. The clogging would give a large increase of the resistance.

As the increase is shown only for the cake of W below 0.05 kg/m^2, it is possible that the clogging would be interrupted after the formation of very thin cake. This phenomenon could happen to the 0.8 μm MF filter, i.e., the increase in average specific cake resistance in Fig. 4 was not by the decrease of the porosity in cake but by the clogging of the filter medium. The discussion about this subject will be continued to the next section.

The experiments as of the previous method were carried out with a filter medium of Toyo 5C and 5A as shown in Fig. 7. The retained diameters of Toyo 5C and 5A media given by the manufacturer are 1 μm and 7 μm, respectively, and the average pore diameters calculated by the method proposed by the author [1998] with the permeation velocity of particle eliminated water, the thickness, and porosity of filter medium were 0.95 μm and 2.80 μm, respectively. The average pore sizes of Toyo 5C and of 0.8 μm MF membrane are similar but the experimental results are somewhat different. The former is composed of a bed of fibers, and the

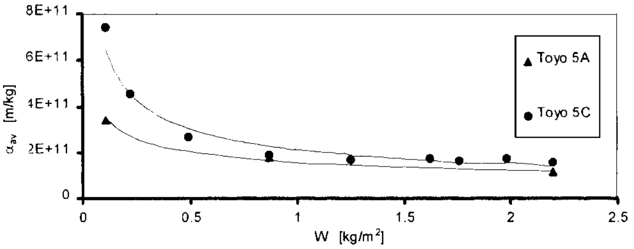

Fig. 7. Measured average specific cake resistances for the various W of $CaCO_3$ suspensions at 0.5 bar Toyo 5C and 5A filter.

latter is a single polymer sheet with numerous holes. The overall tendency of experimental results with fiber filter media (Toyo filters) resembles closely the polymer media (MF filters); the different aspects are as follows.

The increase in average specific cake resistance of polymer media was detected below 0.1 kg/m^2 of W, and that of fiber filters was detected below 0.5 kg/m^2. The fiber media have higher rate of increase in resistance than the polymer media. The structure of fiber media can cause clogging more easily, and can clog until the formation of thicker cake.

With the above experimental results and calculated results, it is difficult to say which one is the more important factor for the increase in average specific cake resistance during the initial period between the cake compression by the "unified theory on solid-liquid separation" and filter medium clogging.

6. Filtration-Permeation with Fine Particle Eliminated Suspension

To reduce the clogging of fine particles to the filter medium or filter cake, we performed the operation as follows to decrease fine particles before "filtration-permeation".

Instead of 1 wt% suspension previously used, 2 wt% suspension was prepared and then precipitated. The solid blanket was not found at this concentration. After the sedimentation of fairly large particles, the pale milky white color of suspension indicates that fine particles are suspended in water. Within 15 minutes only the fine particles were suspended.

At this moment 80% of the suspension was poured out, and then particle-eliminated water was added to the sediment. The mixture was agitated lightly, then precipitated again. This operation was repeated 9 times to reduce fine particles. With the final sediment, suspension for filtration-permeation was prepared. After filtration-permeation of thin cakes, the cake and filter medium was dried and weighed in a balance.

Experimental results with 0.45 μm polymer medium (MF filter) are shown in Fig. 8. We thought that fine particles were eliminated to a certain extent with the 9 times sedimentations. In this case also, the average specific cake resistances of very small W have large values. There was not much difference between the experimental results with and without sedimentation. We do not think that it is solid proof about the absence of clogging at the initial period of filtration. But the experimental result gives the possibility of explaining the large average specific cake resistance at the initial period with the "unified theory on solid-liquid separation".

If the large average specific cake resistance originates from the fine particle migration and clogging at the narrows, there is no reason that this phenomenon could not be prolonged to the permeation period. Then the values of dt/dV must increase during permeation period for the filtration-permeation of very small W. We could not find the above phenomenon in the experiments.

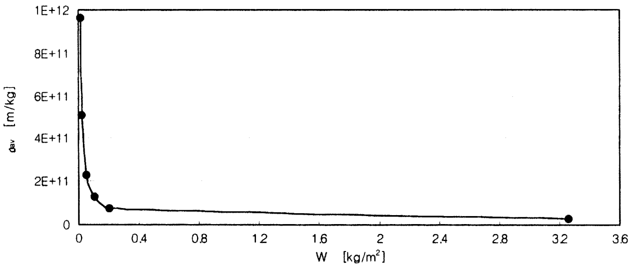

Fig. 8. Measured average specific cake resistances for the various W of fine particle eliminated suspensions at 0.5 bar with 0.45 μm MF filter.

CONCLUSION

The rate of flow, the speed of cake formation, and the rapid change of the rate of filtrate at the initial period of cake filtration are too fast to be measured with conventional filtration apparatus. To solve these problems, an experimental method, "filtration-permeation", with a small area filtration cell was applied to cake filtration. By permeating the particle-eliminated water through very thin cakes which represent the initial period of filtration, average specific cake resistances were measured.

For calcium carbonate suspension, the above method was applied and the following experimental results were obtained. The very thin cakes, which essentially have small values of cake mass per unit filter area W, have large values of average specific cake resistance compared to the comparatively thick cakes. To know the effect of the filter medium on the initial period, several filter media were tested such as 0.8 μm, 0.45 μm, 0.1 μm polymer membranes and fiber filters. All experiments show the large average specific cake resistances for very thin cakes.

Theoretical average specific cake resistances at initial period were calculated by Tiller's conception and the author's "unified theory on solid-liquid separation". By Tiller's approach, the calculated average specific cake resistances at this period were small because of the large porosity due to the small pressure drop across the thin filter cake. This theoretical result is contrary to the experimental results in this study. The calculated average specific cake resistances at the period by the author's concept were larger than the latter period of filtration.

With the new conception the fast velocity due to thin cake thickness causes larger drag force to all of the particles from the first solid layer of cake, and the drag force reduces cake porosity. The reduction of porosity results in large average specific cake resistance.

And the effect of fine particle migration at the initial period of filtration was explored with the same experimental technique for fine particle-reduced suspension by previous sedimentation.

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NOMENCLATURE

ΔP : filtration pressure [Pa]
ΔP_c : pressure drop across cake [Pa]
ΔP_m : pressure drop across filter medium [Pa]
p_s : solid compressive pressure [Pa]
R_m : resistance of filter medium [m^{-1}]
S : mass fraction of solids in suspension which enters into a filter cake [-]
S_c : mass fraction of solids in cake [-]
t : time for filtration [s]
V : filtrate volume per unit filter area [m^3/m^2]
V_o : velocity of fluid out of packed bed [m/s]
W : dry cake mass per unit filter area [kg/m^3]

Greek Letters

α_{av} : average specific cake resistance [m/kg]
$\alpha_{av,f}$: average specific cake resistance by filtration [m/kg]
$\alpha_{av,p}$: average specific cake resistance by permeation [m/kg]
μ : viscosity of filtrate [kg/ms]

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