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DEAN'S FLOW OF AQUEOUS SOLUTION OF POLY(ETHYLENE OXIDE)

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Abstract—In this research the flow of Newtonian and drag reducing fluids through a helical tube, *i.e.*, Dean's flow, was experimentally studied. The primary concern was to investigate the effect of added polymer on the secondary motion caused by the centrifugal force. The polymer chosen in this study was poly(ethylene oxide) with the molecular weights of 300,000, 900,000 and 4,000,000 and the solvent was distilled water. The concentration range was 0 to 100 wppm. The Dean number investigated was in the range of 10 to 20,000. In the case of distilled water, the experimental data were in good agreement with the literature. In the case of polymer solutions, it was found that the secondary motion was supressed as the concentration and molecular weight increased. However, if the molecular weight or concentration exceeded certain values, the effects were saturated. The results were also analyzed from the view point of drag reduction phenomena.

Key words: Drag Reduction, Turbulent Burst, Secondary Motion, Agglomeration

INTRODUCTION

It has been well-known that the pressure drop in a turbulent pipe flow is substantially lowered when a small amount of polymer is added to the flowing solvent. This phenomenon is called drag reduction. Many researches have been directed toward the understanding of the mechanism of drag reduction. It has been a well-known hypothesis that drag reduction is related to the stretching motion in the wall layer [Eckelmann, 1985]. Since very dilute polymer soltuions behave as a non-Newtonian fluid when subjected to elongational flows, the wall layer structure should be different with drag reduction. This is possible because, in the wall layer, the laminar flow along the main flow direction is disturbed by the intermittently and sometimes periodically arising counter rotating eddies and ejections from the wall caused by these eddies. Experimentally, Donohue et al. [1972] has shown that the distance between the low speed streaks becomes larger when the polymer is added. Since the low speed streak is closely related to the existence of vortex pairs, their experimental results confirm that the turbulent bursts are related to drag reduction.

Since the eddying motion and turbulent burst have a large component of extensional motion, the wall layer structure should be affected by the added polymer. Extensional motions caused by eddies or turbulent bursts are difficult to study directly due to the random nature in time and position. To circumbent this difficulty, Dean's flow could be studied which has been known to be similar to turbulent burst [Jones and Davies, 1976]. Dean's flow is steady and therefore more systematic approach is possible. In Dean's flow, a pair of vortices is formed overlapped on the main laminar flow due to the centrifugal force in a curved pipe. Tsang and James [1980] also noted that the Dean's flow was similar to the flow near the wall region in turbulent shear flow in that there existed a counter rotating vortices. When a pair of vortices is formed, the location of the velocity maximum moves outward from the center and the friction loss becomes larger. The formation of the secondary motion was analyzed first by Dean [Berger et al., 1983]. Dean established the theory for small Dean number flow. McConalogue and Srivastava [1968] extended the Dean's solution to large Dean number flow. Wang [1981] pointed out that the coordinate system was not orthogonal when the torsion of the pipe was considered and obtained the asymtotic solution using a regular perturbation technique.

Barns and Walters [1969] first performed the theoretical analysis for non-Newtonian fluids using the third order fluid model. They also performed experimental studies usin gaqueous solution of polyacrylamide and reported that the flow of the polymer solution was different from that of Newtonian fluids. Jones and Davies [1976] studied the flow of dilute solution of polyacrylamide and Kelzan® (xanthan gum) experimentally and showed that the secondary motions were deferred. But until the present time, there have been no studies which consider the effect of molecular weight or concentration of polymer systematically.

In this research experimental studies are performed to investigate the effect of molecular weight and concentration on Dean's flow using aqueous solutions of poly(ethylene oxide) (PEO) which have been widely studied in drag reduction researches. The correlation between the secondary motion in Dean's flow and drag reduction is also investigated. In the next section, we briefly review the theory of Dean's flow and extract relevant parameters. In section 3 experimental procedures are described and in section 4 the results of this research are described.

DEAN'S FLOW

Let us consider a curved pipe wound around a cylinder. The inner radius of the pipe is a and the radius of the cylinder is R. We use the toroidal coordinate system (r', α, θ), where r' is the distance from the center of the coiled pipe, α the angle between the plane of the symmetry and radius vector which originates from the center of the pipe for a given cross section, and θ is the angular distance of the cross section from an arbitrarily chosen position along the center of the coiled pipe. The corre-

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sponding velocity components are (u', v', w'). For a more systematic approach, we introduce the following non-dimensional variables:

$$r=\frac{r'}{a},\ s=\frac{R\theta}{a},\ t=\frac{\bar{W}_0 t'}{a},\ \mathbf{v}=\frac{\mathbf{v}'}{\bar{W}_0},\ p=\frac{p'}{\rho \bar{W}_0},\ \delta=\frac{a}{R} \tag{1}$$

where primed variables are dimensional quantities while unprimed variables are dimensionless ones. In the above equation, t is time, $\mathbf{v}$ is velocity, p is pressure, ρ is density and $\bar{W}_0$ is the mean axial velocity. The governing equations for the flow of Newtonian fluid expressed in the above variables are found in Berger et al. [1983]. The boundary conditions are the no slip condition at the wall and uniform pressure imposed at the entrance. But the latter condition should not be relevant for the fully developed flow.

When the curvature of the coil is small ($\delta \ll 1$), the governing equations for a steady fully developed flow can be expanded by using δ as the perturbation parameter and rescaling (u, v, w) and s as follows:

$$(u,\ v,\ w)=(\delta^{1/2}\tilde{u},\ \delta^{1/2}\tilde{v},\ \tilde{w}),\ s=\delta^{-1/2}z. \tag{2}$$

Then the zeroth order term is the flow through the straight pipe and the first order term is given as follows [Berger et al., 1983]:

$$\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{1}{r}\frac{\partial v}{\partial \alpha}=0 \tag{3}$$

$$u\frac{\partial u}{\partial r}+\frac{v}{r}\frac{\partial u}{\partial \alpha}-\frac{v^2}{r}-w^2\cos\alpha=-\frac{\partial P_1}{\partial r}-\frac{2}{\kappa}\frac{1}{r}\frac{\partial}{\partial \alpha}\left(\frac{\partial v}{\partial r}+\frac{v}{r}-\frac{1}{r}\frac{\partial u}{\partial \alpha}\right) \tag{4}$$

$$u\frac{\partial v}{\partial r}+\frac{v}{r}\frac{\partial v}{\partial \alpha}+\frac{uv}{r}+w^2\sin\alpha=-\frac{1}{r}+\frac{\partial P_1}{\partial \alpha}+\frac{2}{\kappa}\frac{\partial}{\partial r}\left(\frac{\partial v}{\partial r}+\frac{v}{r}-\frac{1}{r}\frac{\partial u}{\partial \alpha}\right) \tag{5}$$

$$u\frac{\partial w}{\partial r}+\frac{v}{r}\frac{\partial w}{\partial \alpha}=-\frac{1}{r}\frac{\partial P_0}{\partial z}+\frac{2}{\kappa}\left[\left(\frac{\partial}{\partial r}+\frac{1}{r}\right)\frac{\partial w}{\partial r}+\frac{1}{r^2}\frac{\partial^2 w}{\partial \alpha^2}\right] \tag{6}$$

where tilde sign(˜) is dropped for the velocity component and κ is the Dean number and defined as follows:

$$\kappa=2\delta^{1/2}Re=\left(\frac{a}{R}\right)^{1/2}\left(\frac{2a\bar{W}_0}{\nu}\right) \tag{7}$$

As seen in the above equations, the flow through a curved pipe is determined by two parameters δ and κ if $\delta \ll 1$ is satisfied. The Dean number has the physical meaning of (inertial force × centrifugal force)$^{1/2}$/(viscous force). Since the inertial and centrifugal force tend to increase the secondary flow while the viscous force tends to decrease the secondary motion, Dean number may be regarded as a measure of the magnitude of the secondary motion. The Dean number defined above is modified from what Dean used originally as the dimensionless number [Berger et al., 1983]. Instead of κ, Dean used

$$K=2\left(\frac{a}{R}\right)\left(\frac{aW_0}{\nu}\right)^2 \tag{8}$$

In the above equation W_0 is the maximum velocity in a straight pipe of the same radius under the same axial pressure gradient.

In the case of Newtonian fluid, the flow rate depends only on K and is given by Dean [Berger et al., 1983]:

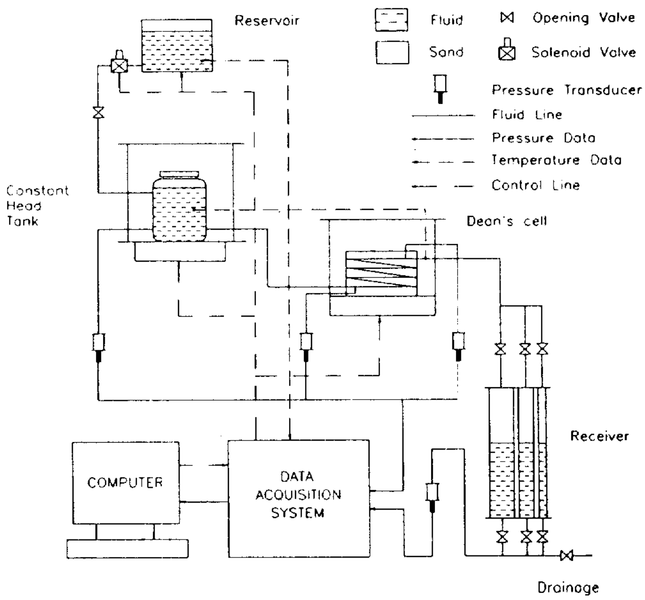

Fig. 1. Schematic diagram of the experimental apparatus.

$$\frac{Q_c}{Q_s}=1-0.0306\left(\frac{K}{576}\right)^2+0.0120\left(\frac{K}{576}\right)^4+\cdots \tag{9}$$

where $\bar{W}=W_0/2$ is the mean velocity, Q_c and Q_s are flow rates for the curved and straight tubes, respectively. This relationship holds for small K/576. For K>576, Barns and Walters [1969] have reported that the following experimental equation fits the experimental data

$$\frac{Q_c}{Q_s}=1-0.175(\log K-2.65) \tag{10}$$

EXPERIMENTAL

The experimental set-up consists of the Dean's flow cell, fluid reservoir, fluid tank for level control and data acquisition system as illustrated in Fig. 1. For the Dean's flow cell, a commercial flexible plastic tubing (1/8 inch Tygon tube) was coiled 7 and 1/4 times on a plexiglas cylinder with a machined helical groove. The actual inner diameter of the tubing was found to be 0.310 cm by weighing the filled distilled water. It was assumed that the cross section of the tubing after coiling was a perfect circle because the wall thickness of the tubing was large and the tubing was coiled along the helical groove. The mean radius of curvature and pitch of the helix were 15.1 cm and 0.64 cm, respectively.

As the solvent doubly distilled water was used. The polymer used in this research was reagent grade poly(ethylene oxide) (PEO) manufactured by Aldrich Chemical Co. The munufacturer supplied molecular weight (MW) of PEO was 300,000, 900,000 and 4,000,000. The polymer was used as received. Polymer solutions were prepared first by preparing master solution of 3,000 ppm. Then the master solution was diluted as desired. The master solution and diluted solutions were prepared by stirring slowly with a magnetic stirrer not to induce shear degradation. The prepared solution was filtered using a glass filter to eliminate large aggregates if any.

For precise temperature control, the reservoir, constant level tank and Dean's cell were placed in separate constant temperature

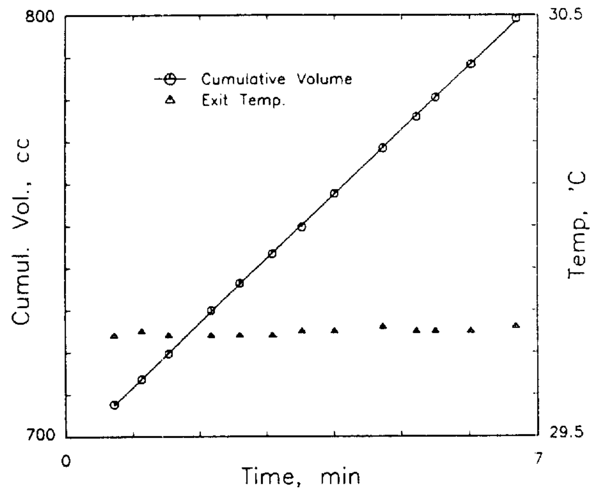

Fig. 2. Temperature variation in the curved tube and the cummulative volume in the receiver tank for 10 ppm aqueous solution of PEO (M.W. 900,000).

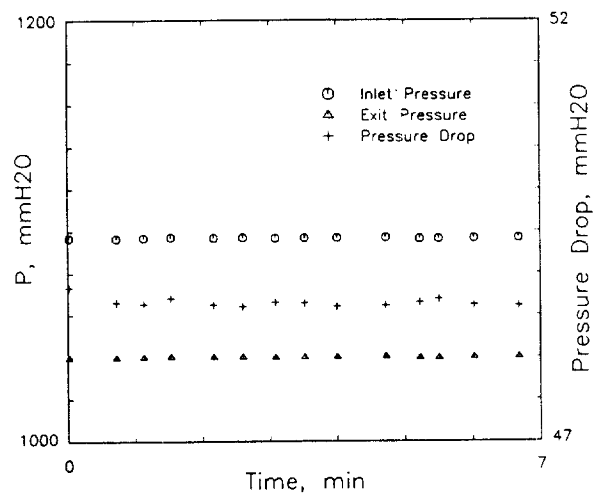

Fig. 3. Pressure variations at the pressure taps in the curved pipe for the flow for 50 ppm aqueous solution of PEO (M.W. 900,000).

chambers which were controlled by the HP3852A data acquisition/control unit. The temperature of the flowing fluid was measured at the exit not to disturb the flow in the Dean's cell. The flow rate was determined by measuring the pressure rise at the bottom of the receiving cylindrical tank for a given interval and then converting to the cumulative volume. All the experimental runs were performed at 30℃.

The pressure drop was measured between the two pressure taps separated by 5 and 1/4 turns by using two pressure transducers connected to the HP3852A. To maintain constant flow rate, the liquid level was controlled by measuring the liquid head with a pressure transducer and controlling a solenoid valve.

In Figs. 2 and 3, a typical set of experimental data is shown. The temperature of the fluid is controlled within ±0.03℃ and the cumulative volume at the exit is linear against time. This means that the flow rate is constant. The pressure drop is also steady and remains within ±0.2 mmH_2O. When the flow rate was large, the variation in the pressure drop was larger than the case shown in Fig. 3. But the relative variation remained small

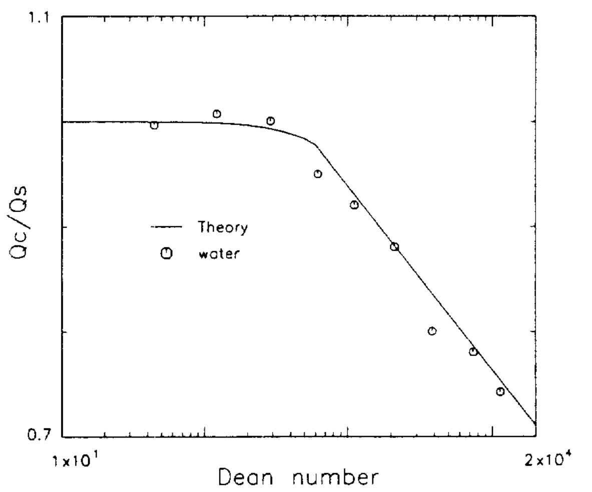

Fig. 4. Q_c/Q_s vs. Dean number for water.

as in Fig. 3.

In calculating the Dean number, the viscosity of the polymer solution was assumed to be independent of shear rate since the polymer solution tested here was very dilute (The increase in zero shear rate viscosity over distilled water was less than 25%). The actual viscosity value of a polymer solution at a given concentration was estimated by using the experimental result of pressure drop vs. flow rate for the small Dean number case since, when the Dean number tends to zero, the flow rate in the coiled tube should be the same as in the straight tube for which the Hagen-Poiseuille equation could be applied. Since PEO is known to be susceptible to mechanical degradation, every effort was made to maintain the same experimental condition.

RESULTS AND DISCUSSION

The flow rate Q_c and viscosity of the polymer solution in this experiment were in the ranges of 0.042-1.0 cm^3/sec and 0.8-1.0 cp, respectively. The Reynolds and Dean numbers are then in the range of 10-1000, and 10-20,000, respectively. It has been reported that the flow in a curved pipe or tube is more stable than that in a straight pipe [Berger et al., 1983]. For example the critical Reynolds number in a curved tube with $a/R = 1/31.9$ was 5000. This corresponds to $K \approx 1.6 \times 10^6$. Since the range of Dean number studied was less than 20,000, the flow should remain laminar in this experimental study.

To investigate the effect of added polymers and to validate the experimental apparatus, distilled water was tested first. In Fig. 4, experimentally determined Q_c/Q_s, is plotted against Dean number together with the Dean's asymptotic solution [Eq. (9)] and Barnes and Walters' equation [Eq. (10)] for comparison. The discontinuity in the derivative of the theoretical curve occurs when the Dean number is 576 since the two equations are patched there. The data obtained for the Newtonian solvent in this research fit well to the Dean's asymptotic solution and Barnes and Walters' result, and hence we confirm that the experimental set up works properly. The slightly lower value than the theoretical curve at the cross-over is also observed in Jones and Davies [1976]. When the Dean number is small, say less than 500, the flow rate for the coiled tube is the same as that for the straight tube. When the Dean number exceeds 600, the flow rate for the

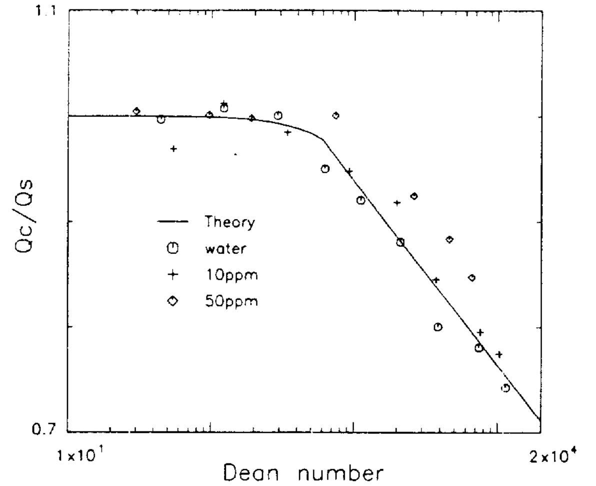

Fig. 5. Q_c/Q_s vs. Dean number for water, PEO (M.W. 900,000) 10 and 50 ppm solutions.

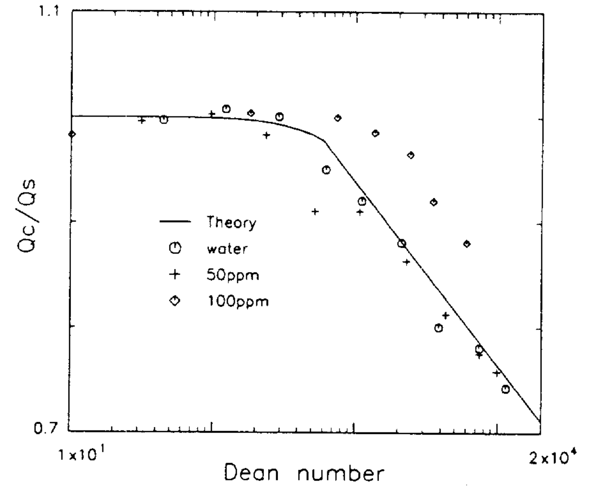

Fig. 7. Q_c/Q_s vs. Dean number for water, PEO (M.W. 300,000) 50 and 100 ppm solutions.

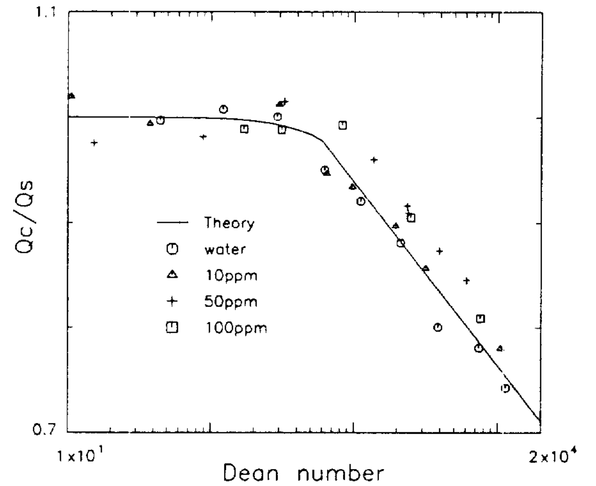

Fig. 6. Q_c/Q_s vs. Dean number for water, PEO (M.W. 4,000,000) 10, 50 and 100 ppm solutions.

coiled tube becomes smaller than that for the straight one and the deviation of Q_c/Q_s from 1 becomes larger as the Dean number increases. This reduction should be caused by the secondary flow in the coiled tube.

Fig. 5 shows a typical experimental result for an aqueous solution of PEO of MW 900,000. When the polymer concentration is 50 ppm and the Dean number is small, the flow rate of the polymer solution is the same as that of the Newtonian solvent in the straight pipe. As the Dean number becomes large, the flow rate in the coiled tube becomes smaller than that in the straight tube as in the case of Newtonian solvent. But the experimetal data are shifted to large Dean number without changing the shape of the curve. The shear rate is in the range of 3-100/sec for this case so that the flow time scale is about 0.01-0.3 sec. The Zimm relaxation time of PEO of MW 900,000 in distilled water is found to be 0.5 millisecond. Since the time scale of the primary flow is much larger than the time scale of the relaxation time of the dilute solution tested here, the primary flow in the axial direction is not much disturbed, and hence the shift is caused by the reduced secondary flow in the polymer solution. When the concentration is 10 ppm, the shape of the curve is the same, but the shift is small and hence the reduction of the secondary flow is small.

Fig. 6 shows that, when the molecular weight is 4,000,000, the shape of the flow curve is the same as when it is 900,000. As the concentration increases from 0 (water data) to 50 ppm, the reduction of secondary motion becomes more conspicuous. But the 100 ppm data appear to be the same as the 50 ppm data within the experimental errors. It is noted that the saturation of reduction for high concentration is analogous to the Virk's maximum drag reduction asymptote [Virk, 1975]. Specifically, in the case of PEO with MW of 1-6 millions, it has been reported that the amount of turbulent drag reduction becomes saturated when the concentration is higher than 50 ppm [Paterson and Abernathy, 1972]. Fig. 7 shows that, when the MW is 300,000, the qualitative nature of the flow curve is the same as before. But in this case, 50 ppm is not concentrated enough to show any visible non-Newtonian effect. The slightly lower value than the theoretical curve at the cross-over is already observed in the Newtonian case of Fig. 4. The relatively larger deviation appears to be due to experimental errors. It is probable that the experimental errors were caused by the oxidative degradation of polymer solutions and/or aging even though every effort was made to maintain the same experimental conditions inpreparing the polymer solution and controlling the equipment. In the 100 ppm case, the cross-over Dean number is larger than those of the high molecular weight cases. The cross-over Dean number should be determined by the relaxation time of the solution. Specifically, for larger relaxation time, the non-Newtonian effect should be larger. But as described below, it is not possible at the present report that how large it should be.

To examine the effect of molecular weight more clearly, in Fig. 8, the data for 50 ppm solution with varying MW are plotted. The data shown here are redrawn from Figs. 5, 6 and 7. In this figure, it is noted that molecular weight effect is saturated for MW of 900,000 or larger.

If we compare the experimental results on the Dean's flow of PEO solutions with the drag reduction data reported in literatures, we may note two independent analogues between these

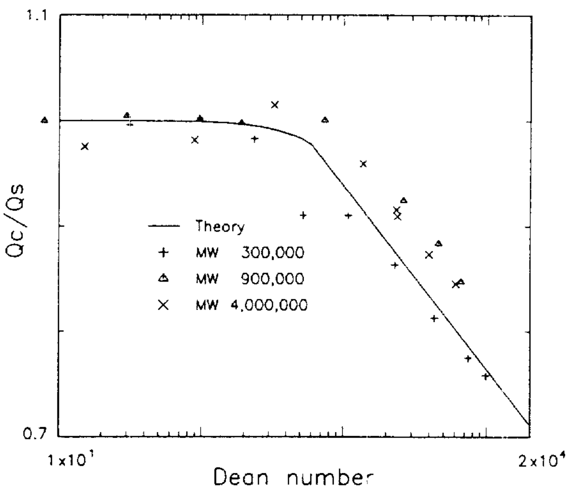

Fig. 8. Q_c/Q_s vs. Dean number for 50 ppm solutions of varying molecular weights.

two systems. The reduction of secondary flow in the Dean's flow and drag in the turbulent flow increases as the concentration of polymer increases to a certain point but the effect becomes saturated for higher concentrations. For the molecular weight, the similar phenomenon is observed. From the two analogues, it, is presumable that in turbulent pipe flows, bursting in the drag reducing fluid is less frequent and/or weaker than in Newtonian fluids even though a quantitative analysis for the reduction has not been possible.

Using the experimental result of Jones and Davies [1976], Tsang and James [1980] determined the shear and extensional rates at the cross-over Dean number and estimated the extensional stress by using the Rouse and network models. They argued that the Rouse model was inappropriate and perhaps a network model should be required. Noting that the range of the Dean number for which the secondary flow is deferred is within the same range of Tsang and James' analysis, the dilute solutions of PEO considered here may follow the network model. This fact may be arised because the polymer is not distributed uniformly during dilution of the master solution even after the filtration and remains as agglomerates. The same phenomena was also reported in Dunlop and Cox [1977] for the drag reduction experiment. Hinch and Elata [1979] also proposed that fresh polymer solution was heterogeneous. The formation of agglomerates or networks even in the case of the dilute solution has been also reported by Brennen and Gadd [1967], Layec-Raphlen and Layec [1985] and Morgan and McCormick [1990].

In this report, it is not pursued to give the quantitative analysis for the experimental result since no firmly established constitutive equation exists for the flow of dilute polymer solutions that exhibit such complex behaviors rheologically. It should be beyond the scope of the present experimental research. Qualitatively, it is obvious that networks are easily formed in the cases of higher molecular weight polymers and more concentrated solutions. It appears that the networks suppress the formation of secondary motions. The fact that networks or agglomerates are formed even in the dilute solution implies that the drag reduction mechanism should not be approached from the view point of continuum mechanics only.

SUMMARY

In this research experimental studies are performed to investigate the effect of molecular weight and concentration on Dean's flow using aqueous solutions of poly(ethylene oxide) (PEO) which have been widely studied in drag reduction researches. In the case of distilled water, the experimental data were in good agreement with the literature. For the case of polymer solutions, it was found that the secondary motion was suppressed as the concentration and molecular weight increased. However, if the molecular weight or concentration exceeded certain values, the effects became saturated. This saturation is similar to the maximum drag reduction asymptote. The reduced secondary motion cannot be predicted by using the conventional dumbbell models. Therefore, it is estimated that the dilute solutions of PEO considered here follow the network model.

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NOMENCLATURE

a : radius of tube
K : Dean number [Eq. (8)]
p : pressure
s : arc length
t : time
u, v, w : velocity components in the toroidal coordinate system
v : velocity
W_0 : maximum velocity along the primary flow direction
$\bar{W}_0$: average velocity along the primary flow direction
z : scaled s
Q_c : flow rate in the curved tube
Q_s : flow rate in the straight tube

Greek Letters

α : toroidal coordinate
δ : a/R
κ : Dean number [Eq. (7)]
θ : toroidal coordinate
ρ : density

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