總 說
REVIEW

電磁場下에서의 氣體의 分子運動論

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Kinetic Theory Under External Fields

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Abstract

Kinetic theory based upon the classical Boltzmann equation is applied to the analysis of (1) Senftleben-Beenakker (SB) effects upon the thermal conductivity and viscosity, (2) thermomagnetic torque (TMT) effect of polyatomic gases. New predictions are shown to be in better agreement with recent TMT experiments. Collision integrals are calculated from SB experimental data.

I. Introduction

In 1930 Senftleben[1] discovered a slight decrease of viscosity of paramagnetic oxygen gas in a magnetic field. Later it is found by Beenakker and his colleagues at Leiden[2] that even diamagnetic gases such as nitrogen had the same effects. Since then external field (magnetic or electric) effects upon the thermal conductivity and viscosity of polyatomic gases (Senftleben-Beenakker Effects) have been studied extensively both theoretically and experimentally, and it is reviewed recently by McCourt and Beenakker[3].

In the meantime Scott et al[4]. discovered a remarkable phenomenon in 1967. Conducting Einstein-de Hass measurements, they found that a heated cylinder experienced a torque when suspended in a polyatomic gas and acted upon by an axially directed magnetic field and a radial temperature gradient. Later experiments provide an evidence that the gas around the cylinder is set into a very slow swirling motion[5].

This thermomagnetic torque seems to vary linearly with the temperature gradient and it is inversely proportional to the pressure except at pressures so low that there also is evidence of free molecular flow. The direction of the torque is reversed if either the direction of the magnetic field H or of the temperature gradient is reversed.

There is a field strength, characteristic of the gas, at which the torque reaches a maximum value. No effect has been observed so far with the noble gases and the torque does not appear to depend upon the character of the cylinder surface. It is noteworthy that Yun[6] recently raised the question of torque invariance under a temperature reversal. It was clarified immediately that the thermomagnetic torque (TMT) cannot be explained as a simple transfer of molecular angular momentum. Levi and Beenakker[7] then suggested that it might arise from a second order stress which was proportional to $\nabla\nabla T$ and that this stress could be computed from the second order Chapman-Enskog kinetic theory. Leiden group has extended the theory based upon the Waldmann-Snider equation[8]. By exploiting the correlation between the first order SB effects and the second order TMT they evaluated the torque which differed from experiments by factors of two or three. Employing the same but more direct method Park and Dahler have obtained better agreement with recent experiments[9].

The Leiden group and Waldmann as well paid attention to the possible importance of boundary effects at the gas-solid interface. Park and Dahler made a thorough study of the boundary conditions and of the temperature and velocity profiles appropriate to the torque experiments. They found that boundary effects are more important than thought previously, with the torque due to field induced thermal creep accounting for approximately one third of the total torque and in the opposite direction. This fact is confirmed later by Waldmann and his colleagues[10].

The present article summarizes the kinetic theory method of predicting TMT by invoking the more generallized collision cross section. It is shown that systematic derivation of TMT formula gives the same result of Levi, McCourt, and Beenakker, but that it renders a more versatile tool to explore other effects.

II. Kinetic Theory and Results

A. Senftleben-Beenakker Effects

It can be shown that field effects are directly related to the anisotropy in angular momentum J. The influence of magnetic field, for example, is brought out by the partial destruction of the anisotropies, which is caused by the magnetically induced Larmor precession of J. Whether based upon the classical Boltzmann equation[11] or upon the quantum mechanical Waldmann-Snider equation[12], kinetic theory studies revealed that the leading terms containing angular momentum J in trial expansions are $W\ [J]^{(2)}$ for the thermal conductivity and $[J]^{(2)}$ for the viscosity. If we truncate the expansions right after these terms, theoretical results can be written as

$$\frac{\Delta\lambda_{11}}{\lambda_0}=-\rho_\lambda f(\xi_\lambda), \qquad (1)$$

$$\frac{\Delta\lambda_\perp}{\lambda_0}=-\rho_\lambda\left[\frac{1}{2}f(\xi_\lambda)+f(2\xi_\lambda)\right] \qquad (2)$$

$$\frac{\lambda_{tr}}{\lambda_0}=-\rho_\lambda\left[\frac{1}{2}g(\xi_\lambda)+g(2\xi_\lambda)\right] \qquad (3)$$

and

$$\frac{\Delta\eta_1}{\eta_0}=0 \qquad (4)$$

$$\frac{\Delta\eta_2}{\eta_0}=-\rho_\eta\frac{1}{2}f(2\xi_\eta). \qquad (5)$$

$$\frac{\Delta\eta_3}{\eta_0}=-\rho_\eta f(\xi_\eta), \qquad (6)$$

$$\frac{\eta_4}{\eta_0}=-\rho_\eta g(2\xi_\eta). \qquad (7)$$

$$\frac{\eta_5}{\eta_0}=+\rho_\eta g(\xi_\eta). \qquad (8)$$

where λ_{11}, $\lambda_\perp$, and λ_{tr} denotes thermal conductivity componets parallel, perpendicular, and transverse (perpendicular both to λ_{11} and $\lambda_\perp$) to the magnetic field direction, respectively, $\eta_i's$ are five independent shear viscosity components defined by de Groot and Mazur[13]. Furthermore,

$$\rho_\lambda=\frac{\left[\left\{\sigma(1001)+\frac{1}{r}\sigma\binom{1010}{1001}\right\}\sigma\binom{1010}{1200}\right.}{\left[\sigma(1001)+\frac{2}{r}\sigma\binom{1010}{1001}+\frac{1}{r^2}\sigma(1010)\right]} \qquad (9)$$
$$\frac{\left.-\left\{\sigma\binom{1010}{1001}+\frac{1}{r}\sigma(1010)\right\}\sigma\binom{1001}{1200}\right]^2}{\sigma_0(1200)\left[\sigma(1010)\sigma(1001)-\sigma^2\binom{1010}{1001}\right]}$$

with $\gamma^2=5/2$ for diatomic gases, and

$$\rho_\eta=\frac{\sigma^2\binom{2000}{0200}}{\sigma(2000)\sigma(0200)} \qquad (10)$$

$$f(x)=\frac{x^2}{1+x^2}\ \text{even function} \qquad (11)$$

$$g(x)=\frac{x}{1+x^2}\ \text{odd function} \qquad (12)$$

and for diamagnetic gases such as N_2 and CO,

$$\xi_\lambda=\frac{1}{\langle v\rangle\sigma_0(1200)}\ \frac{kTg\mu_N}{\hbar}\ \frac{H}{p} \qquad (13)$$

$$\xi_\eta=\frac{1}{\langle v\rangle\sigma(0200)}\ \frac{kTg\mu_N}{\hbar}\ \frac{H}{p} \qquad (14)$$

with g denoting the molecular g-factor, μ_N the nuclear magnetion, $\langle v\rangle=\sqrt{\frac{8kT}{\pi\mu}}$ with μ the reduced mass, and finally λ_0 and η_0 are field-free ($H=0$) thermal conductivity and viscosity, respectively. The genera-

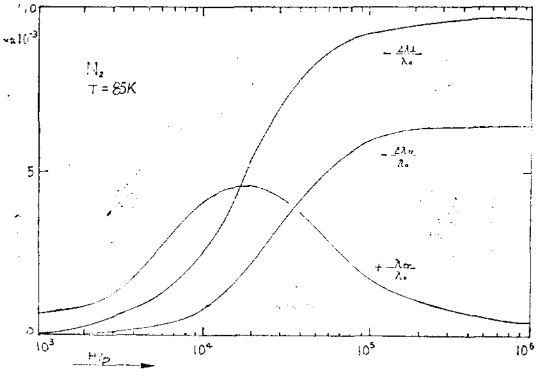

Fig. 1. Thermal conductivity change of N_2 under magnetic field at 85°K.

lized collision cross sections are defined in the reference 14.

Typical experimental data is given below for the case of nitrogen gas[15,16,17])

The collision cross sections which appear in Eqns. (9) and (10) can be computed from these experiments at a specified temperature[18]). Once the temperature dependence of σ's is constructed graphically, it may be very easy to predict other transport properties at any temperature. (See the following section.)

B. Thermomagnetic Torque

Park and Dahler[9]) showed that TMT is the sum of a thermal creep contribution τ^B, which depneds upon the translational portion of the thermal conductivity coefficient λ_{tr} and of the thermal stress contribution τ^C which is proportional to $\triangle\triangle T$.

$$\tau=\tau^B+\tau^C, \tag{15}$$

$$\tau^C=4\pi\overline{\mho}C_1 \tag{16}$$

where

with $T=\varDelta T/l_n(\gamma_0/\gamma_i)$, and C_1 representing the second order phenomenological coefficient which correlates the second order pressure with $\triangle\triangle T$. C_1 is evaluated in Reference11 from the general second order Chapman-Enskog equation.

Since $\sigma^2\binom{1010}{1001}\Big/\sigma(1010)\sigma(1001) \doteqdot O(10^{-2})$ for diamagnetic gases such as N_2 and CO, we may neglect $\sigma^2\binom{1010}{1001}$ compared to $\sigma(1010)\times\sigma(1001)$. When we invoke this approximation to the formula for the coefficient C_1, we have

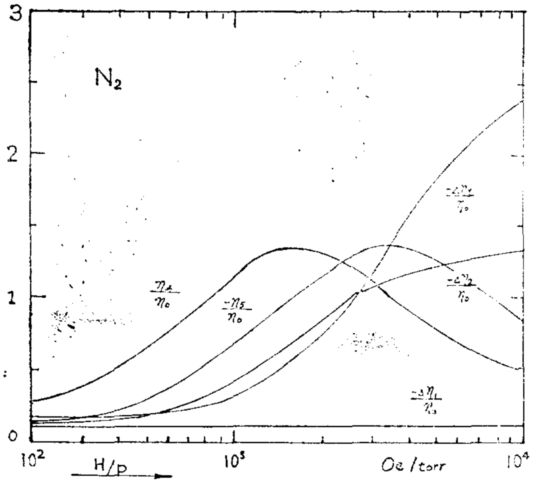

Fig. 2. Viscosity change of N_2 under magnetic field at room temperature.

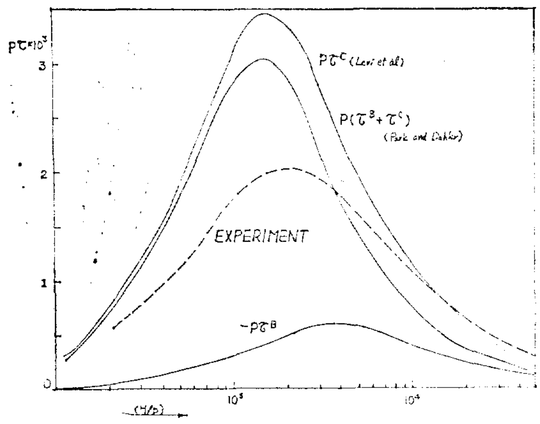

Fig. 3. Thermomagnetic torque compnted from theories.

$$\tau^B=-\frac{1}{2}\tau^{(0)} \tag{17}$$

$$\tau^C=\tau^{(0)}+\tau^* \tag{18}$$

where

$$p\tau^{(0)}=\frac{8}{5}\pi h\,\eta_0\lambda_0{}^{(TR)}\,\overline{\mho}\Big[-A\frac{1}{2}g(\xi_\lambda)-Ag(2\xi_\lambda)\Big] \tag{19}$$

and

$$p\tau^*=\frac{8}{5}\pi h\eta_0\lambda_0{}^{(TR)}\overline{\mho}\Big[-Bg(2\,\xi_\lambda)+Cg(2\,\mu\xi_\lambda)\Big] \tag{20}$$

with

$$\mu=\frac{\sigma_0(1200)}{\sigma(0200)}$$

Here constants A, B, and C are functions of collision cross section σ's all of which appear in the first order SB effects. Levi et al[8] have already derived the same formul a which can be written as

$$p\tau^C=\frac{8}{5}\pi h\eta_0\lambda_0^{(TR)}\ \widetilde{\mathbb{O}}\left[-A\frac{1}{2}g(\xi_\lambda)-(B+A)g(2\xi_\lambda)+Cg(2\mu\xi_\lambda)\right] \quad (21)$$

Actually the dominant factor in TMT formula is C, and we can immediately see that park and Dahler's prediction of the torque is less in magnitude than Levi et al's.

It is noteworthy that, when the temperature dependence of collision cross seetions obtained from SB effects is applied to the torque formula, Adair's[19] exprimental results of TMT temperature dependence can be reproduced.

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